

Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & -1 & 2 \\ 2 & m & m \\ 3 & 0 & 3 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} 3 & m & 0 \\ m & 2 & 2 \\ -m & 3 & 4 \end{vmatrix} = 3m^2 - 9m + 6 = 0 \implies m = 1, \quad m = 2$$

Si $m = 1$ o $m = 2 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq 2 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 0 & -1 & 2 \\ 2 & 0 & 0 \\ 3 & 0 & 3 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} 0 & 1/2 & 0 \\ -1 & -1 & 2/3 \\ 0 & -1/2 & 1/3 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $X - AX - B = C$. Donde

$$A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 4 \\ -1 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 3 & 7 \\ 1 & 0 \end{pmatrix}$$

Solución:

$$X - AX - B = C \implies X = (I - A)^{-1}(C + B)$$

$$I - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 0 & -1 \end{pmatrix}, \quad (I - A)^{-1} = \begin{pmatrix} -1/2 & 1/2 \\ 0 & -1 \end{pmatrix}$$

$$C + B = \begin{pmatrix} 3 & 7 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 11 \\ 0 & 3 \end{pmatrix}$$

$$X = (I - A)^{-1}(C + B) = \begin{pmatrix} -1/2 & 1/2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 & 11 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -4 \\ 0 & -3 \end{pmatrix}$$

Problema 3 Resolver, utilizando las propiedades de los determinantes, calcular:

$$\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix}$$

Solución:

$$\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{vmatrix} x+3 & x+3 & x+3 & x+3 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} =$$

$$(x+3) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = \begin{bmatrix} C_1 \\ C_2 - C_1 \\ C_3 - C_1 \\ C_4 - C_1 \end{bmatrix} = (x+3) \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & x-1 & 0 & 0 \\ 1 & 0 & x-1 & 0 \\ 1 & 0 & 0 & x-1 \end{vmatrix} =$$

$$(x+3) \begin{vmatrix} x-1 & 0 & 0 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} = (x-1)^3(x+3)$$