

Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & 0 & -m & 2 \\ 2 & 1 & m & m \\ 4 & 3 & 5 & -m \end{pmatrix}$$

Calcular el rango de A para los diferentes valores de m .

Solución:

$$|A_1| = \begin{vmatrix} m & 0 & -m \\ 2 & 1 & m \\ 4 & 3 & 5 \end{vmatrix} = 3m(1-m) = 0 \implies m = 0, m = 1$$

$$|A_2| = \begin{vmatrix} m & 0 & 2 \\ 2 & 1 & m \\ 4 & 3 & -m \end{vmatrix} = -4(m^2 - 1) = 0 \implies m = 1, m = -1$$

$$|A_3| = \begin{vmatrix} m & -m & 2 \\ 2 & m & m \\ 4 & 5 & -m \end{vmatrix} = -m^3 - 11m^2 - 8m + 20 = 0 \implies m = 1, m = -22, m = -10$$

$$|A_4| = \begin{vmatrix} 0 & -m & 2 \\ 1 & m & m \\ 3 & 5 & -m \end{vmatrix} = -2(2m^2 + 3m - 5) \implies m = 1, m = -5/2$$

Si $m \neq 1 \implies \text{Rango}(A) = 3$.

Cuando $m = 1 \implies \text{Rango}(A) = 2$, ya que el menor $\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = 2 \neq 0$.

Problema 2 Dada la matriz

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

Calcular A^n y en particular A^{100}

Solución:

$$A^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -3 & 0 & 1 \end{pmatrix}$$

$$A^n = \begin{cases} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -n & 0 & 1 \end{pmatrix} & \text{si } n \text{ impar} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -n & 0 & 1 \end{pmatrix} & \text{si } n \text{ par} \end{cases}$$

$$A^{100} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -100 & 0 & 1 \end{pmatrix}$$

Problema 3 Resolver el siguiente sistema matricial:

$$\begin{cases} X - 3Y = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} \\ X + Y = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} \end{cases}$$

Solución:

$$\begin{cases} X - 3Y = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} \\ X + Y = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} \end{cases} \implies \begin{cases} X = \begin{pmatrix} 7/4 & 3/4 \\ 11/4 & 1/2 \end{pmatrix} \\ Y = \begin{pmatrix} 1/4 & -3/4 \\ 1/4 & 1/2 \end{pmatrix} \end{cases}$$

Problema 4 calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix}$$

Solución:

$$\text{LLamamos } X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}:$$

$$AX = XA \implies \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix} \implies$$

$$\begin{pmatrix} a + 3c & b + 3d \\ -c & -d \end{pmatrix} = \begin{pmatrix} a & 3a - b \\ c & 3c - d \end{pmatrix} \implies \begin{cases} a + 3c = a \\ b + 3d = 3a - b \\ -c = c \\ -d = 3c - d \end{cases} \implies \begin{cases} c = 0 \\ b = \frac{3a - 3d}{2} \end{cases}$$

$$\text{LLamamos } X = \begin{pmatrix} a & \frac{3a-3d}{2} \\ 0 & d \end{pmatrix}$$