

Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & 2 & -m & 2 \\ 2 & m & 2 & -1 \\ 3 & 3 & 1 & m \end{pmatrix}$$

Calcular el rango de A para los diferentes valores de m .

Solución:

$$|A_1| = \begin{vmatrix} m & 2 & -m \\ 2 & m & 2 \\ 3 & 3 & 1 \end{vmatrix} = 4m^2 - 12m + 8 = 0 \implies m = 1, \quad m = 2$$

$$|A_2| = \begin{vmatrix} m & 2 & 2 \\ 2 & m & -1 \\ 3 & 3 & m \end{vmatrix} = m^3 - 7m + 6 = 0 \implies m = 1, \quad m = 2, \quad m = -3$$

$$|A_3| = \begin{vmatrix} m & -m & 2 \\ 2 & 2 & -1 \\ 3 & 1 & m \end{vmatrix} = 4m^2 + 4m - 8 = 0 \implies m = 1, \quad m = -2$$

$$|A_4| = \begin{vmatrix} 2 & -m & 2 \\ m & 2 & -1 \\ 3 & 1 & m \end{vmatrix} = m^3 + 9m - 10 = 0 \implies m = 1$$

Si $m \neq 1 \implies \text{Rango}(A) = 3$.

Cuando $m = 1 \implies \text{Rango}(A) = 2$, ya que el menor $\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3 \neq 0$.

Problema 2 Dada la matriz

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Calcular A^n y en particular A^{101}

Solución:

$$A^1 = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$A^4 = A^3 \cdot A = \begin{pmatrix} -1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^n = \begin{cases} \begin{pmatrix} 1 & 0 & -n \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} & \text{si } n \text{ par} \\ \begin{pmatrix} -1 & 0 & n \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} & \text{si } n \text{ impar} \end{cases}$$

$$A^{101} = A = \begin{pmatrix} -1 & 0 & 101 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Problema 3 Resolver el siguiente sistema

$$\begin{cases} 2X - Y = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix} \\ X + Y = \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \end{cases}$$

Solución:

$$\begin{cases} 2X - Y = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix} \\ X + Y = \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \end{cases} \implies \begin{cases} X = \begin{pmatrix} 5/3 & 1 \\ -1/3 & 1 \end{pmatrix} \\ Y = \begin{pmatrix} 1/3 & 1 \\ 4/3 & -2 \end{pmatrix} \end{cases}$$

Problema 4 Calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$$

Solución:

LLamamos $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$AX = XA \implies \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} \implies$$

$$\begin{pmatrix} a & b \\ 2a - c & 2b - d \end{pmatrix} = \begin{pmatrix} a + 2b & -b \\ c + 2d & -d \end{pmatrix} \implies \begin{cases} a = a + 2b \implies b = 0 \\ b = -b \implies b = 0 \\ 2a - c = c + 2d \implies a = c + d \\ 2b - d = -d \implies b = 0 \end{cases}$$

$$\text{Luego } X = \begin{pmatrix} c + d & 0 \\ c & d \end{pmatrix}.$$