

Problema 1 Sea la matriz

$$A = \begin{pmatrix} 1 & m-1 & 0 & m \\ m & -1 & 1 & -1 \\ 1 & -5 & m & -8 \end{pmatrix}$$

Calcular el rango de A para los diferentes valores de m .

Solución:

$$|A_1| = \begin{vmatrix} 1 & m-1 & 0 \\ m & -1 & 1 \\ 1 & -5 & m \end{vmatrix} = -m^3 + m^2 + 4 = 0 \implies m = 2$$

$$|A_2| = \begin{vmatrix} 1 & m-1 & m \\ m & -1 & -1 \\ 1 & -5 & -8 \end{vmatrix} = 3m^2 - 8m + 4 = 0 \implies m = 2, m = 2/3$$

$$|A_3| = \begin{vmatrix} 1 & 0 & m \\ m & 1 & -1 \\ 1 & m & -8 \end{vmatrix} = m^3 - 8 = 0 \implies m = 2$$

$$|A_4| = \begin{vmatrix} m-1 & 0 & m \\ -1 & 1 & -1 \\ -5 & m & -8 \end{vmatrix} = 8 - 4m = 0 \implies m = 2$$

Si $m \neq 2 \implies \text{Rango}(A) = 3$.

Cuando $m = 2 \implies \text{Rango}(A) = 2$, ya que el menor $\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 2 \neq 0$.

Problema 2 Dada la matriz

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Calcular A^n y en particular A^{100}

Solución:

$$A^1 = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$A^3 = A^2 \cdot A = I \cdot A = A$$

$$A^n = \begin{cases} A & \text{si } n \text{ impar} \\ I & \text{si } n \text{ par} \end{cases}$$

$$A^{100} = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Problema 3 Resolver el siguiente sistema matricial:

$$\begin{cases} X - 5Y = \begin{pmatrix} 1 & 5 \\ -2 & -1 \end{pmatrix} \\ X + Y = \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix} \end{cases}$$

Solución:

$$\begin{cases} X - 5Y = \begin{pmatrix} 1 & 5 \\ -2 & -1 \end{pmatrix} \\ X + Y = \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix} \end{cases} \implies \begin{cases} X = \begin{pmatrix} 11/6 & 5/3 \\ 3 & 2/3 \end{pmatrix} \\ Y = \begin{pmatrix} 1/6 & -2/3 \\ 1 & 1/3 \end{pmatrix} \end{cases}$$

Problema 4 calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$$

Solución:

LLamamos $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$AX = XA \implies \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} \implies$$

$$\begin{pmatrix} -a + 2c & -b + 2d \\ a & b \end{pmatrix} = \begin{pmatrix} -a + b & 2a \\ -c + d & 2c \end{pmatrix} \implies \begin{cases} -a + 2c = -a + b \\ -b + 2d = 2a \\ a = -c + d \\ b = 2c \end{cases} \implies \begin{cases} a = -c + d \\ b = 2c \end{cases}$$

LLamamos $X = \begin{pmatrix} -c + d & 2c \\ c & d \end{pmatrix}$