RELACIÓN DE EJERCICIOS DE INTEGRALES.

1. $\int x^5 dx$

$$\int x^5 dx = \frac{x^{5+1}}{5+1} + k = \frac{x^6}{6} + k$$

2. $\int (x + \sqrt{x}) dx$

$$\int (x + \sqrt{x}) dx = \int (x + x^{1/2}) dx = \frac{x^{1+1}}{1+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + k = \frac{x^2}{2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{x^2}{2} + \frac{2\sqrt{x^3}}{3} + k =$$

$$= \frac{x^2}{2} + \frac{2x\sqrt{x}}{3} + k$$

3. $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx$ Sol: $6\sqrt{x} - \frac{1}{10} x^2 \sqrt{x} + k$

$$\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx = \int (3x^{-\frac{1}{2}} - \frac{1}{4} x^{\frac{3}{2}}) dx = 3 \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{1}{4} \cdot \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + k = 3 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{4} \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + k =$$

$$= 6\sqrt{x} - \frac{1}{2} \frac{\sqrt{x^5}}{5} + k = 6\sqrt{x} - \frac{1}{10} x^2 \sqrt{x} + k$$

4. $\int \frac{x^2 dx}{\sqrt{x}}$ Sol: $\frac{2}{5} x^2 \sqrt{x} + k$

$$\int \frac{x^2 dx}{\sqrt{x}} = \int \frac{x^2}{x^{\frac{1}{2}}} dx = \int x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + k = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + k = \frac{2\sqrt{x^5}}{5} + k = \frac{2x^2\sqrt{x}}{5} + k$$

5. $\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx$ Sol: $-\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + k$

$$\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx = \int (x^{-2} + 4x^{-\frac{3}{2}} + 2) dx = \frac{x^{-2+1}}{-2+1} + 4 \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + 2x + k =$$

$$= \frac{x^{-1}}{-1} + 4 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 2x + k = -\frac{1}{x} - 8 \cdot \frac{1}{x^{\frac{1}{2}}} + 2x + k = -\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + k$$

6. $\int \frac{dx}{\sqrt[4]{x}}$ Sol: $\frac{4}{3}\sqrt[4]{x^3}+k$

$$\int \frac{dx}{\sqrt[4]{x}} = \int x^{-\frac{1}{4}}dx = \frac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1}+k = \frac{x^{\frac{3}{4}}}{\frac{3}{4}}+k = \frac{4x^{\frac{3}{4}}}{3}+k = \frac{4}{3}\sqrt[4]{x^3}+k$$

7. $\int\left(x^2+\frac{1}{\sqrt[3]{x}}\right)^2 dx$ Sol: $\frac{x^5}{5}+\frac{3}{4}x^2\sqrt[3]{x^2}+3\sqrt[3]{x}+k$

$$\int\left(x^2+\frac{1}{\sqrt[3]{x}}\right)^2 dx = \int\left(x^2+x^{-\frac{1}{3}}\right)^2 dx = \int(x^4+2x^{\frac{5}{3}}+x^{-\frac{2}{3}}).dx = \frac{x^5}{5}+2\cdot\frac{x^{\frac{8}{3}}}{\frac{8}{3}}+\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+K =$$

$$=\frac{1}{5}\cdot x^5+\frac{3}{4}\cdot x^{\frac{8}{3}}+3.x^{\frac{1}{3}}+K = \frac{1}{5}\cdot x^5+\frac{3}{4}\cdot\sqrt[3]{x^8}+3.\sqrt[3]{x}+K =$$

$$=\frac{1}{5}\cdot x^5+\frac{3}{4}\cdot x^2.\sqrt[3]{x^2}+3.\sqrt[3]{x}+K$$

8. $\int \cos^3 x\cdot \operatorname{sen} x\cdot dx$ Sol: $-\frac{\cos^4 x}{4}+k$

$$\int \cos^3 x\cdot \operatorname{sen} x\cdot dx = -\int \underbrace{\cos^3 x}_{f^3}\cdot\underbrace{(-\operatorname{sen} x)}_{f'}\cdot dx = \frac{\cos^4 x}{4}+k$$

9. $\int x\sqrt{x^2+1}\cdot dx$ Sol: $\frac{1}{3}\sqrt{(x^2+1)^3}+k$

$$\int x\sqrt{x^2+1}\cdot dx = \frac{1}{2}\int \underbrace{2x}_{f'}\cdot\underbrace{(x^2+1)^{\frac{1}{2}}}_{f^{1/2}}dx = \frac{1}{2}\cdot\frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}}+k = \frac{\sqrt{(x^2+1)^3}}{3}+k$$

10. $\int \frac{xdx}{\sqrt{2x^2+3}}$ Sol: $\frac{1}{2}\sqrt{2x^2+3}+k$

$$\int \frac{xdx}{\sqrt{2x^2+3}} = \frac{1}{4}\int \underbrace{4x}_{f'}\cdot\underbrace{(2x^2+3)^{-\frac{1}{2}}}_{f^{-1/2}}dx = \frac{1}{4}\cdot\frac{(2x^2+3)^{\frac{1}{2}}}{\frac{1}{2}}+k = \frac{1}{2}\sqrt{2x^2+3}+k$$ o

$$\int \frac{xdx}{\sqrt{2x^2+3}} = \frac{2}{4}\int\frac{4xdx}{2\sqrt{2x^2+3}} = \left\{\int\frac{f'}{2\sqrt{f}}dx = \sqrt{f}+k\right\} = \frac{1}{2}\sqrt{2x^2+3}+k$$

11. $\int \frac{x^2dx}{\sqrt{x^3+1}}$ Sol: $\frac{2}{3}\sqrt{x^3+1}+k$

$$\int \frac{x^2dx}{\sqrt{x^3+1}} = \int x^2(x^3+1)^{-\frac{1}{2}}dx = \frac{1}{3}\int 3x^2(x^3+1)^{-\frac{1}{2}}dx = \frac{1}{3}\cdot\frac{(x^3+1)^{\frac{1}{2}}}{\frac{1}{2}}+k = \frac{2}{3}\sqrt{x^3+1}+k$$

$$\int \frac{x^2dx}{\sqrt{x^3+1}} = \frac{2}{3}\int \frac{3x^2dx}{2\sqrt{x^3+1}} = \left\{\int \frac{f'}{2\sqrt{f}}dx = \sqrt{f}+k\right\} = \frac{2}{3}\sqrt{x^3+1}+k$$

12. $\int \frac{\cos x}{\text{sen}^2 x}dx$ Sol: $-\frac{1}{\text{sen}\, x}+k$

$$\int \frac{\cos x}{\text{sen}^2 x}dx = \int \underbrace{\cos x}_{f'}\cdot\underbrace{\text{sen}^{-2} x}_{f^{-2}}dx = \frac{\text{sen}^{-1} x}{-1}+k = -\frac{1}{\text{sen}\, x}+k$$

13. $\int x\cdot(x^2+1)^4dx$ Sol: $\frac{(x^2+1)^5}{10}+k$

$$\int x\cdot(x^2+1)^4dx = \frac{1}{2}\int \underbrace{2x}_{f'}\cdot\underbrace{(x^2+1)^4}_{f^4}dx = \frac{1}{2}\cdot\frac{(x^2+1)^5}{5}+k = \frac{(x^2+1)^5}{10}+k$$

14. $\int \frac{\text{sen}\, x}{\cos^3 x}\cdot dx$ Sol: $\frac{1}{2\cos^2 x}+k$

$$\int \frac{\text{sen}\, x}{\cos^3 x}\cdot dx = -\int \underbrace{-\text{sen}\, x}_{f'}\cdot\underbrace{\cos^{-3} x}_{f^{-3}}dx = -\frac{\cos^{-2} x}{-2}+k = \frac{1}{2\cos^2 x}+k$$

15. $\int \frac{\text{tg}\, x}{\cos^2 x}\cdot dx$ Sol: $\frac{\text{tg}^2 x}{2}+k$

$$\int \frac{\text{tg}\, x}{\cos^2 x}\cdot dx = \int \underbrace{\text{tg}\, x}_{f^1}\cdot\underbrace{\frac{1}{\cos^2 x}}_{f'}\cdot dx = \frac{\text{tg}^2 x}{2}+k$$

16. $\int \frac{1}{\cos^2 x\sqrt{\text{tg}\, x-1}}\cdot dx$ Sol: $2\sqrt{\text{tg}\, x-1}+k$

$$\int \frac{1}{\cos^2 x\sqrt{\text{tg}\, x-1}}\cdot dx = \int \frac{1}{\cos^2 x}\cdot(\text{tg}\, x-1)^{-\frac{1}{2}}\cdot dx = \frac{(\text{tg}\, x-1)^{\frac{1}{2}}}{\frac{1}{2}}+k = 2\sqrt{\text{tg}\, x-1}+k$$

17. $\displaystyle\int \frac{\mathrm{L}(x+1)}{x+1}dx$ Sol: $\displaystyle\frac{\mathrm{L}^2(x+1)}{2}+k$

$$\int \frac{\mathrm{L}(x+1)}{x+1}dx=\int \underbrace{\mathrm{L}(x+1)}_{f^1}\cdot\underbrace{\frac{1}{x+1}}_{f'}dx=\frac{\mathrm{L}^2(x+1)}{2}+k$$

18. $\displaystyle\int \frac{\cos x}{\sqrt{2\operatorname{sen} x+1}}dx$ Sol: $\sqrt{2\operatorname{sen} x+1}+k$

$$\int \frac{\cos x}{\sqrt{2\operatorname{sen} x+1}}dx=\frac{1}{2}\int \underbrace{2\cos x}_{f'}\underbrace{(2\operatorname{sen} x+1)^{-\frac{1}{2}}}_{f^{-1/2}}dx=\frac{1}{2}\cdot\frac{(2\operatorname{sen} x+1)^{\frac{1}{2}}}{\frac{1}{2}}+k=\sqrt{2\operatorname{sen} x+1}+k$$

19. $\displaystyle\int \frac{\sqrt{\operatorname{tg} x+1}}{\cos^2 x}\cdot dx$ Sol: $\displaystyle\frac{2}{3}\sqrt{(\operatorname{tg} x+1)^3}+k$

$$\int \frac{\sqrt{\operatorname{tg} x+1}}{\cos^2 x}\cdot dx=\int (\operatorname{tg} x+1)^{\frac{1}{2}}\cdot\frac{1}{\cos^2 x}\cdot dx=\frac{(\operatorname{tg} x+1)^{\frac{3}{2}}}{\frac{3}{2}}+k=\frac{2}{3}\sqrt{(\operatorname{tg} x+1)^3}+k$$

20. $\displaystyle\int \frac{\cos 2x}{(2+3\operatorname{sen} 2x)^3}dx$ Sol: $\displaystyle-\frac{1}{12}\frac{1}{(2+3\operatorname{sen} 2x)^2}+k$

$$\int \frac{\cos 2x}{(2+3\operatorname{sen} 2x)^3}dx=\frac{1}{6}\int 6\cos 2x\cdot(2+3\operatorname{sen} 2x)^{-3}dx=\frac{1}{6}\cdot\frac{(2+3\operatorname{sen} 2x)^{-2}}{-2}+k=$$

$$=-\frac{1}{12}\frac{1}{(2+3\operatorname{sen} 2x)^2}+k$$

21. $\displaystyle\int \frac{\operatorname{sen} 3x}{\sqrt[3]{\cos^4 3x}}dx$ Sol: $\displaystyle\frac{1}{\sqrt[3]{\cos 3x}}+k$

$$\int \frac{\operatorname{sen} 3x}{\sqrt[3]{\cos^4 3x}}dx=-\frac{1}{3}\int \underbrace{-3\operatorname{sen} 3x}_{f'}\underbrace{\cos^{-\frac{4}{3}} 3x}_{f^{-4/3}}dx=-\frac{1}{3}\cdot\frac{\cos^{-\frac{1}{3}} 3x}{-\frac{1}{3}}+k=\frac{1}{\sqrt[3]{\cos 3x}}+k$$

22. $\int \frac{\arcsin x\, dx}{\sqrt{1-x^2}}$ Sol: $\frac{\arcsin^2 x}{2}+k$

$$\int \frac{\arcsin x\, dx}{\sqrt{1-x^2}} = \int \arcsin x \frac{1}{\sqrt{1-x^2}} dx = \frac{\arcsin^2 x}{2}+k$$

23. $\int \frac{\arccos^2 x\, dx}{\sqrt{1-x^2}}$ Sol: $-\frac{\arccos^3 x}{3}+k$

$$\int \frac{\arccos^2 x\, dx}{\sqrt{1-x^2}} = -\int \arccos^2 x \frac{-1}{\sqrt{1-x^2}} dx = -\frac{\arccos^3 x}{3}+k$$

24. $\int \frac{x}{x^2+1} dx$ Sol: $\frac{1}{2}\mathrm{Ln}(x^2+1)+k$

$$\int \frac{x}{x^2+1} dx = \frac{1}{2}\int \frac{2x}{x^2+1} dx = \left\{\int \frac{f'}{f} dx = \mathrm{Ln}|f|+k\right\} = \frac{1}{2}\mathrm{Ln}(x^2+1)+k$$

25. $\int \frac{dx}{3x-7}$ Sol: $\frac{1}{3}\mathrm{L}\,|3x-7|+k$

$$\int \frac{dx}{3x-7} = \frac{1}{3}\int \frac{3}{3x-7} \cdot dx = \left\{\int \frac{f'}{f} dx = \mathrm{Ln}|f|+k\right\} = \frac{1}{3}\mathrm{Ln}\,|3x-7|+k$$

26. $\int \frac{dx}{5-2x}$ Sol: $-\frac{1}{2}\mathrm{L}\,|5-2x|+k$

$$\int \frac{dx}{5-2x} = -\frac{1}{2}\int \frac{-2}{5-2x} \cdot dx = \left\{\int \frac{f'}{f} dx = \mathrm{Ln}|f|+k\right\} = -\frac{1}{2}\mathrm{Ln}\,|5-2x|+k$$

27. $\int \frac{x+1}{x^2+2x+3} dx$ Sol: $\frac{1}{2}\mathrm{L}\,|\,x^2+2x+3\,|+k$

$$\int \frac{x+1}{x^2+2x+3} dx = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x+3} dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+3} dx = \frac{1}{2}\mathrm{L}\,|\,x^2+2x+3\,|+k$$

28. $\int \mathrm{tg}\, x\, dx$ Sol: $-\mathrm{Ln}\,|\cos x|+k$

$$\int \mathrm{tg}\, x \cdot dx = \int \frac{\mathrm{sen}\, x}{\cos x} \cdot dx = -\int \frac{-\mathrm{sen}\, x}{\cos x} \cdot dx = -\mathrm{Ln}\,|\cos x| + K$$

29. $\int \mathrm{tg}\, 2x\, dx$ Sol: $-\frac{1}{2}\mathrm{L}\, |\cos 2x| + k$

$$\int \mathrm{tg}\, 2x \cdot dx = \int \frac{\mathrm{sen}\, 2x}{\cos 2x} \cdot dx = -\frac{1}{2}\int \frac{-2\mathrm{sen}\, 2x}{\cos 2x} \cdot dx = -\frac{1}{2}\mathrm{Ln}\, |\cos 2x| + K$$

30. $\int \mathrm{ctg}\, x\, dx$ Sol: $\mathrm{Ln}\, |\mathrm{sen}\, x| + k$

$$\int \mathrm{ctg}\, x \cdot dx = \int \frac{\cos x}{\mathrm{sen}\, x} \cdot dx = \mathrm{Ln}\, |\mathrm{sen}\, x| + K$$

31. $\int \frac{dx}{\mathrm{ctg}\, 3x}$ Sol: $-\frac{1}{3}\mathrm{L}\, |\cos 3x| + k$

$$\int \frac{dx}{\mathrm{ctg}\, 3x} = \int \mathrm{tg}\, 3x \cdot dx = \int \frac{\mathrm{sen}\, 3x}{\cos 3x} \cdot dx = -\frac{1}{3}\int \frac{-3\mathrm{sen}\, 3x}{\cos 3x} \cdot dx = -\frac{1}{3}\mathrm{Ln}\, |\cos 3x| + K$$

32. $\int \mathrm{ctg}\, \frac{x}{3} dx$ Sol: $3\mathrm{L}\left| \mathrm{sen}\, \frac{x}{3} \right| + k$

$$\int \mathrm{ctg}\, \frac{x}{3} \cdot dx = \int \frac{\cos \frac{x}{3}}{\mathrm{sen}\, \frac{x}{3}} \cdot dx = 3\int \frac{\frac{1}{3}\cos \frac{x}{3}}{\mathrm{sen}\, \frac{x}{3}} \cdot dx = 3 \cdot \mathrm{Ln}\, \left|\mathrm{sen}\, \frac{x}{3}\right| + K$$

33. $\int \left(\mathrm{tg}\, 4x - \mathrm{ctg}\, \frac{x}{4}\right) dx$ Sol: $\mathrm{Sol:} -\frac{1}{4}\mathrm{Ln}\, |\cos 4x| - 4\mathrm{Ln}\, \left|\mathrm{sen}\, \frac{x}{4}\right| + k$

$$\int \left(\mathrm{tg}\, 4x - \mathrm{ctg}\, \frac{x}{4}\right) dx = \int \left(\frac{\mathrm{sen}\, 4x}{\cos 4x} - \frac{\cos \frac{x}{4}}{\mathrm{sen}\, \frac{x}{4}}\right) dx = \int \frac{\mathrm{sen}\, 4x}{\cos 4x} dx - \int \frac{\cos \frac{x}{4}}{\mathrm{sen}\, \frac{x}{4}} dx =$$

$$= -\frac{1}{4}\int \frac{-4\mathrm{sen}\, 4x}{\cos 4x} dx - 4\int \frac{\frac{1}{4}\cos \frac{x}{4}}{\mathrm{sen}\, \frac{x}{4}} dx = -\frac{1}{4}\mathrm{Ln}\, |\cos 4x| - 4\,\mathrm{Ln}\, \left|\mathrm{sen}\, \frac{x}{4}\right| + k$$

34. $\int \frac{\cos x}{2\mathrm{sen}\, x + 3} dx$ Sol: $\frac{1}{2}\mathrm{Ln}\, (2\mathrm{sen}\, x + 3) + k$

$$\int \frac{\cos x}{2\mathrm{sen}\, x + 3} dx = \frac{1}{2}\int \frac{2\cos x}{2\mathrm{sen}\, x + 3} dx = \frac{1}{2}\mathrm{Ln}\, (2\mathrm{sen}\, x + 3) + k$$

35. $\displaystyle\int \frac{dx}{(1+x^2)\operatorname{arc\,tg} x}$ Sol: $\mathrm{Ln}\,|\operatorname{arc\,tg} x| + k$

$$\int \frac{dx}{(1+x^2)\operatorname{arc\,tg} x} = \int \frac{\frac{1}{1+x^2}}{\operatorname{arc\,tg} x}dx = \mathrm{Ln}\,|\operatorname{arc\,tg} x| + k$$

36. $\displaystyle\int \frac{\cos 2x}{2+3\operatorname{sen} 2x}dx$ Sol: $\dfrac{1}{6}\mathrm{Ln}\,|2+3\operatorname{sen} 2x| + k$

$$\int \frac{\cos 2x}{2+3\operatorname{sen} 2x}dx = \frac{1}{6}\int \frac{6\cos 2x}{2+3\operatorname{sen} 2x}dx = \frac{1}{6}\mathrm{Ln}\,|2+3\operatorname{sen} 2x| + k$$

37. $\displaystyle\int e^{2x}dx$ Sol: $\dfrac{1}{2}e^{2x} + k$

$$\int e^{2x}dx = \frac{1}{2}\int 2e^{2x}dx = \left\{\int f'e^{f}dx = e^{f} + k\right\} = \frac{1}{2}e^{2x} + k$$

38. $\displaystyle\int e^{\frac{x}{2}}dx$ Sol: $2e^{\frac{x}{2}} + k$

$$\int e^{\frac{x}{2}}dx = 2\int \frac{1}{2}e^{\frac{x}{2}}dx = \left\{\int f'e^{f}dx = e^{f} + k\right\} = 2e^{\frac{x}{2}} + k$$

39. $\displaystyle\int e^{\operatorname{sen} x}\cos x\,dx$ Sol: $e^{\operatorname{sen} x} + k$

$$\int e^{\operatorname{sen} x}\cos x\,dx = \left\{\int f'e^{f}dx = e^{f} + k\right\} = e^{\operatorname{sen} x} + k$$

40. $\displaystyle\int a^{x^2}\cdot x\cdot dx$ Sol: $\dfrac{a^{x^2}}{2\,\mathrm{L}\,a} + k$

$$\int a^{x^2}x\cdot dx = \frac{1}{2\mathrm{Ln}\,a}\int \underbrace{a^{x^2}2x\cdot \mathrm{Ln}\,a}_{D(a^{x^2})}dx = \frac{1}{2\mathrm{Ln}\,a}a^{x^2} + k$$

41. $\displaystyle\int e^{\frac{x}{a}}dx$ Sol: $ae^{\frac{x}{a}} + k$

$$\int e^{\frac{x}{a}}dx = a\int \frac{1}{a}e^{\frac{x}{a}}dx = ae^{\frac{x}{a}} + k$$

42. $\int \left(e^{2x}\right)^2 dx$ Sol: $\frac{1}{4}e^{4x}+k$

$$\int \left(e^{2x}\right)^2 dx = \int e^{4x}dx = \frac{1}{4}\int 4e^{4x}dx = \frac{1}{4}e^{4x}+k$$

43. $\int 5^x e^x dx$ Sol: $\frac{5^x e^x}{\text{Ln }5+1}+k$

$$\int 5^x e^x dx = \int (5e)^x dx = \frac{1}{\text{Ln}(5e)}\int (5e)^x \text{Ln}(5e)dx = \frac{1}{\text{Ln}(5e)}(5e)^x + k = \frac{5^x e^x}{\text{Ln }5+1}+k$$

44. $\int \left(e^{5x}+a^{5x}\right)dx$ Sol: $\frac{1}{5}\left(e^{5x}+\frac{a^{5x}}{\text{L }a}\right)+k$

$$\int \left(e^{5x}+a^{5x}\right)dx = \frac{1}{5}\int 5e^{5x}dx + \frac{1}{5\text{ Ln }a}\int 5a^{5x}\text{Ln }a\,dx = \frac{1}{5}e^{5x}+\frac{1}{5\text{ Ln }a}\cdot a^{5x}+k =$$

$$= \frac{1}{5}\left(e^{5x}+\frac{a^{5x}}{\text{L }a}\right)+k$$

45. $\int e^{x^2+4x+3}(x+2)dx$ Sol: $\frac{1}{2}e^{x^2+4x+3}+k$

$$\int e^{x^2+4x+3}(x+2)dx = \frac{1}{2}\int e^{x^2+4x+3}2(x+2)dx = \frac{1}{2}e^{x^2+4x+3}+k$$

46. $\int \frac{e^x}{3+4e^x}dx$ Sol: $\frac{1}{4}\text{Ln }(3+4e^x)+k$

$$\int \frac{e^x}{3+4e^x}dx = \frac{1}{4}\int \frac{4e^x}{3+4e^x}dx = \frac{1}{4}\text{Ln }(3+4e^x)+k$$

47. $\int \cos 5x dx$ Sol: $\frac{1}{5}\text{sen }5x+k$

$$\int \cos 5x dx = \frac{1}{5}\int 5\cos 5x dx = \left\{\int f'(x)\cdot \cos f(x)dx = \text{sen } f(x)+k\right\} = \frac{1}{5}\text{sen}5x+k$$

48. $\int \mathbf{tg}^2\, x\, dx$ Sol: $\mathbf{tg}\, x - x + k$

Por trigonometría sabemos que $\mathbf{tg}^2 x + 1 = \sec^2 x \quad \Rightarrow \quad \mathbf{tg}^2 x = \sec^2 x - 1,$ entonces

$$\int \mathbf{tg}^2\, x\, dx = \int (\sec^2 x - 1) dx = \int \sec^2 x\, dx - \int dx = \mathbf{tg}\, x - x + k$$

49. $\int \frac{\cos(\mathrm{Ln}(x))}{x} dx$ Sol: $\mathrm{sen}\, (\mathrm{Ln}(x)) + k$

$$\int \frac{\cos(\mathrm{Ln}(x))}{x} dx = \quad \cos(\mathrm{Ln}(x)) \frac{1}{x} dx = \mathrm{sen}\, (\mathrm{Ln}(x)) + k$$

50. $\int \frac{x}{\sqrt{1-x^4}} dx$ Sol: $\frac{1}{2} \mathrm{arc\,sen}\, x^2 + k$

$$\int \frac{x}{\sqrt{1-x^4}} dx = \int \frac{x}{\sqrt{1-(x^2)^2}} dx = \left\{ \int \frac{f'(x)}{\sqrt{1-(f(x))^2}} dx = \begin{cases} \mathrm{arc\,sen}\, f(x) + k \\ -\arccos\, f(x) + k \end{cases} \right\} =$$

$$= \frac{1}{2} \int \frac{2x}{\sqrt{1-(x^2)^2}} dx = \left\{ \int \frac{f'(x) dx}{\sqrt{1-(f(x))^2}} \right\} = \frac{1}{2} \mathrm{arc\,sen}(x^2) + k$$

51. $\int \frac{dx}{\sqrt{1-4x^2}}$ Sol: $\frac{1}{2} \mathrm{arc\,sen}(2x) + k$

$$\int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{1-(2x)^2}} = \frac{1}{2} \int \frac{2dx}{\sqrt{1-(2x)^2}} = \left\{ \int \frac{f'(x) dx}{\sqrt{1-(f(x))^2}} \right\} = \frac{1}{2} \mathrm{arc\,sen}(2x) + k$$

52. $\int \frac{dx}{\sqrt{9-4x^2}}$ Sol: $\frac{1}{2} \mathrm{arc\,sen}(\frac{2x}{3}) + k$

$$\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{9(1-\frac{4x^2}{9})}} = \int \frac{dx}{3\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \frac{1}{3} \int \frac{dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \frac{1}{3} \cdot \frac{3}{2} \int \frac{\frac{2}{3} dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} =$$

$$= \frac{1}{2} \int \frac{\frac{2}{3} dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \left\{ \int \frac{f'(x) dx}{\sqrt{1-(f(x))^2}} \right\} = \frac{1}{2} \mathrm{arc\,sen}(\frac{2x}{3}) + k$$

53. $\int \frac{x - \text{arctg}\ x}{1+x^2}dx$ Sol: $\frac{1}{2}\text{Ln}(1+x^2) - \frac{1}{2}(\text{arctg}\ x)^2 + k$

$$\int \frac{x - \text{arctg}\ x}{1+x^2}dx = \int \frac{x}{1+x^2}dx - \int \frac{\text{arctg}\ x}{1+x^2}dx = \frac{1}{2}\int \frac{2x}{1+x^2}dx - \int \text{arctg}\ x\ \frac{1}{1+x^2}dx =$$

$$= \left\{\int \frac{f'(x)}{f(x)}dx - \int f^1 \cdot f'dx\right\} = \frac{1}{2}\text{Ln}(1+x^2) - \frac{1}{2}(\text{arctg}\ x)^2 + k$$

54. $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx$ Sol: $\frac{4}{3}\sqrt{(1+\sqrt{x})^3} + k$

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = \int \frac{1}{\sqrt{x}}\sqrt{1+\sqrt{x}}\,dx = 2\int \frac{1}{2\sqrt{x}}\left(1+\sqrt{x}\right)^{\frac{1}{2}}dx = \left\{\int f^{\frac{1}{2}} \cdot f'dx\right\} =$$

$$= 2 \cdot \frac{\left(1+\sqrt{x}\right)^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{4}{3}\sqrt{(1+\sqrt{x})^3} + k$$

Veamos como podemos realizar esta misma integral por el método de <u>*sustitución o cambio de variable.*</u>

Haciendo el cambio $1+\sqrt{x} = t$, calculamos ***dx***: $\frac{1}{2\sqrt{x}}dx = dt \Rightarrow dx = 2\sqrt{x}dt$ y sustituimos en nuestra integral:

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = \int \frac{\sqrt{t}}{\sqrt{x}} \cdot 2\sqrt{x}dt = 2\int \sqrt{t}\ dt = 2\int t^{\frac{1}{2}}dt = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{4}{3}t^{\frac{3}{2}} + k = \frac{4}{3}\sqrt{t^3} + k =$$

una vez realizada la integral hay que deshacer el cambio de variable y volver a la variable ***x,*** con lo que nos quedará:

$$= \frac{4}{3}\sqrt{(1+\sqrt{x})^3} + k$$

55. $\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}dx$ Sol: $4\sqrt{1+\sqrt{x}} + k$

$$\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}dx = \int \frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+\sqrt{x}}}dx = 2\int \frac{1}{2\sqrt{x}}\left(1+\sqrt{x}\right)^{-\frac{1}{2}}dx = \left\{\int f^{-\frac{1}{2}} \cdot f'dx\right\} =$$

$$= 2 \cdot \frac{\left(1+\sqrt{x}\right)^{\frac{1}{2}}}{\frac{1}{2}} + k = 4\sqrt{1+\sqrt{x}} + k$$

Veamos como podemos realizar esta misma integral por el método de sustitución o cambio de variable.

Haciendo el cambio $1+\sqrt{x} = t$, calculamos dx: $\frac{1}{2\sqrt{x}}dx = dt \Rightarrow dx = 2\sqrt{x}dt$ y sustituimos en nuestra integral:

$$\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}dx = \int \frac{1}{\sqrt{x}\sqrt{t}} \cdot 2\sqrt{x}dt = 2\int \frac{1}{\sqrt{t}}\,dt = 2\int t^{-\frac{1}{2}}dt = 2\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + k = 4t^{\frac{1}{2}} + k = 4\sqrt{t} + k =$$

una vez realizada la integral hay que deshacer el cambio de variable y volver a la variable ***x*,** con lo que nos quedará:

$$= 4\sqrt{1+\sqrt{x}} + k$$

56. $\int x\sqrt{x-1}\,dx$ Sol: $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + k$

Hacemos la sustitución $x-1 = t^2 \quad \Rightarrow \quad x = t^2 + 1$

Calculamos la diferencial de *x*: $dx = 2t.dt$ y sustituimos en la integral que deseamos calcular. Tendremos:

$$\int x.\sqrt{x-1} \cdot dx = \int (t^2+1).\sqrt{t^2} \cdot 2tdt = 2\int (t^2+1).t^2.dt = 2\int (t^4+t^2).dt = 2\left(\frac{t^5}{5} + \frac{t^3}{3}\right) + k =$$

$$= \frac{2}{5} \cdot t^5 + \frac{2}{3} \cdot t^3 + k = \frac{2}{5} \cdot (x-1)^{\frac{5}{2}} + \frac{2}{3} \cdot (x-1)^{\frac{3}{2}} + k$$

57. $\int x(5x^2-3)^7\,dx$ Sol: $\frac{1}{80}(5x^2-3)^8 + k$

Directamente:

$$\int x(5x^2-3)^7\,dx = \frac{1}{10}\int 10x(5x^2-3)^7\,dx = \left\{\int f^7 \cdot f'dx\right\} = \frac{1}{10} \cdot \frac{(5x^2-3)^8}{8} + k =$$

$$= \frac{1}{80}(5x^2-3)^8 + k$$

Por sustitución:

Hacemos $5x^2-3=t \Rightarrow 10xdx=dt \Rightarrow dx=\frac{dt}{10x}$ y sustituimos en nuestra integral

$$\int x(5x^2-3)^7\,dx=\int xt^7\frac{dt}{10x}=\frac{1}{10}\int t^7dt=\frac{1}{10}\cdot\frac{t^8}{8}+k=\frac{1}{80}(5x^2-3)^8+k$$

58. $\int x(2x+5)^{10}dx$ Sol: $\frac{1}{4}\left[\frac{(2x+5)^{12}}{12}-\frac{5(2x+5)^{11}}{11}\right]+k$

Por sustitución:

Hacemos $2x+5=t \Rightarrow x=\frac{t-5}{2} \Rightarrow dx=\frac{1}{2}dt$ y sustituimos en nuestra integral

$$\int x(2x+5)^{10}dx=\int\frac{t-5}{2}\cdot t^{10}\cdot\frac{1}{2}dt=\frac{1}{4}\int(t-5)t^{10}dt=\frac{1}{4}\int(t^{11}-5t^{10})dt=$$

$$=\frac{1}{4}\left[\frac{t^{12}}{12}-5\cdot\frac{t^{11}}{11}\right]+k=\frac{1}{4}\left[\frac{(2x+5)^{12}}{12}-\frac{5(2x+5)^{11}}{11}\right]+k$$

59. $\int xe^x dx$ Sol: $e^x(x-1)+k$

Por el método de _integración por partes:_

$$\int xe^xdx=\left\{\begin{matrix}u=x\Rightarrow du=dx\\ dv=e^xdx\Rightarrow v=e^x\end{matrix}\right\}=xe^x-\int e^xdx=xe^x-e^x+k=(x-1)e^x+k$$

60. $I=\int(x^2-3x+5)e^xdx$ Sol: $e^x(x^2-5x+10)+k$

Por el método de integración por partes:

$$I=\int(x^2-3x+5)e^xdx=\left\{\begin{matrix}u=x^2-3x+5\Rightarrow du=(2x-3)dx\\ dv=e^xdx\Rightarrow v=e^x\end{matrix}\right\}=$$

$$=(x^2-3x+5)\ \ e\ \ -\int e^x(2x-3)dx=$$

La integral que nos ha quedado es del mismo tipo que la que pretendemos calcular, por lo que nuevamente aplicaremos el método de integración de partes:

Hacemos $\left\{\begin{matrix}u=2x-3\Rightarrow du=2dx\\ dv=e^xdx\Rightarrow v=e^x\end{matrix}\right\}$ y sustituimos:

$$I=(x^2-3x+5)e^x-\int e^x(2x-3)dx=(x^2-3x+5)e^x-\left[(2x-3e^x-\int 2e^x dx\right]=$$

$$=(x^2-3x+5)e^x-(2x-3)e^x+2\int e^x dx=(x^2-3x+5)e^x-(2x-3)e^x+2e^x+k=$$

$$=\left[(x^2-3x+5)-(2x-3)+2\right]e^x+k=e^x(x^2-5x+10)+k$$

61. $\int x\,\mathrm{Ln}(x)dx$ Sol: $\frac{1}{2}x^2\left(\mathrm{Ln}(x)-\frac{1}{2}\right)+k$

Por el método de integración por partes:

$$\int x\,\mathrm{Ln}(x)dx=\left\{\begin{matrix} u=\mathrm{Ln}(x)\Rightarrow du=\frac{1}{x}dx \\ dv=xdx\Rightarrow v=\frac{1}{2}x^2 \end{matrix}\right\}=\frac{1}{2}x^2\,\mathrm{Ln}(x)-\int\frac{1}{2}x^2\cdot\frac{1}{x}dx=$$

$$=\frac{1}{2}x^2\,\mathrm{Ln}(x)-\frac{1}{2}\int x\,dx=\frac{1}{2}x^2\,\mathrm{Ln}(x)-\frac{1}{2}\cdot\frac{1}{2}x^2+k=\frac{1}{2}x^2\left(\mathrm{Ln}(x)-\frac{1}{2}\right)+k$$

62. $\int e^{-x}\cos x dx$ Sol: $\frac{1}{2}e^{-x}(\operatorname{sen}x-\cos x)+k$

$$I=\int e^{-x}\cos xdx=\left\{\begin{matrix} u=e^{-x}\Rightarrow du=-e^{-x}dx \\ dv=\cos xdx\Rightarrow v=\operatorname{sen}x \end{matrix}\right\}=e^{-x}\operatorname{sen}x-\int -e^{-x}\operatorname{sen}xdx=$$

$$=e^{-x}\operatorname{sen}x+\int e^{-x}\operatorname{sen}xdx$$

Al aplicar el método de partes nos ha quedado una integral del mismo tipo que la que pretendemos calcular, por lo que volvemos a aplicar el mismo método. En ella hacemos:

$$u=e^{-x}\quad\Rightarrow\quad du=-e^{-x}dx$$
$$dv=\operatorname{sen}x\,dx\quad\Rightarrow\quad v=-\cos x$$

Sustituyendo en la expresión anterior nos queda:

$$I=e^{-x}\operatorname{sen}x+\int e^{-x}\operatorname{sen}xdx=e^{-x}\operatorname{sen}x+\left[-\cos x\cdot e^{-x}-\int -\cos x\cdot(-e^{-x})dx\right]=$$

$$=e^{-x}\operatorname{sen}x-\cos x\cdot e^{-x}-\int\cos x\cdot e^{-x}dx$$

es decir, volvemos a la misma integral que pretendemos calcular. Entonces:

$$I = e^{-x}\text{sen}\,x - \cos x \cdot e^{-x} - I \quad \Rightarrow \quad 2I = e^{-x}\text{sen}\,x - \cos x \cdot e^{-x} \quad \Rightarrow \quad I = \frac{e^{-x}(\text{sen}\,x - \cos x)}{2}$$

En consecuencia:

$$\boldsymbol{I = \int e^{-x} \cos x dx = \frac{e^{-x}(\text{sen}\,x - \cos x)}{2} + k}$$

63. $\int \mathbf{Ln}(1-x)\,dx$ Sol: $-x-(1-x)\,\mathbf{Ln}(1-x)+k$

$$\int \mathbf{Ln}(1-x)\,dx = \left\{ \begin{array}{c} u = \mathbf{Ln}(1-x) \Rightarrow du = \dfrac{-1}{1-x}dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x\mathbf{Ln}(1-x) - \int x \cdot \frac{-1}{1-x}dx =$$

$$= x\mathbf{Ln}(1-x) - \int \frac{-x}{1-x}dx = x\mathbf{Ln}(1-x) - \int \frac{1-x-1}{1-x}dx = x\mathbf{Ln}(1-x) - \int \left(1 + \frac{-1}{1-x}\right)dx =$$

$$= x\mathbf{Ln}(1-x) - \left(x + \mathbf{Ln}(1-x)\right) + k = x\mathbf{Ln}(1-x) - x - \mathbf{Ln}(1-x) + k =$$

$$= -x - (1-x)\,\mathbf{Ln}(1-x) + k$$

64. $\int x^n\,\mathbf{Ln}(x)\,dx$ Sol: $\dfrac{x^{n+1}}{n+1}\left(\mathbf{Ln}(x) - \dfrac{1}{n+1}\right) + k$

Por el método de integración por partes:

$$\int x^n\,\mathbf{Ln}(x)\,dx = \left\{ \begin{array}{c} u = \mathbf{Ln}(x) \Rightarrow du = \dfrac{1}{x}dx \\ dv = x^n dx \Rightarrow v = \dfrac{1}{n+1}x^{n+1} \end{array} \right\} = \frac{1}{n+1}x^{n+1}\,\mathbf{Ln}(x) - \int \frac{1}{n+1}x^{n+1} \cdot \frac{1}{x}dx =$$

$$= \frac{1}{n+1}x^{n+1}\,\mathbf{Ln}(x) - \frac{1}{n+1}\int x^n\,dx = \frac{1}{n+1}x^{n+1}\,\mathbf{Ln}(x) - \frac{1}{n+1} \cdot \frac{1}{n+1}x^{n+1} + k =$$

$$= \frac{x^{n+1}}{n+1}\left(\mathbf{Ln}(x) - \frac{1}{n+1}\right) + k$$

65. $\int \arcsin x\, dx$ Sol: $x \arcsin x + \sqrt{1-x^2} + k$

Hacemos el siguiente cambio: $\left\{\begin{matrix} u = \arcsin x \\ dv = dx \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} du = \dfrac{1}{\sqrt{1-x^2}} \cdot dx \\ v = x \end{matrix}\right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\int \arcsin x.dx = x.\arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx = x.\arcsin x - \int x.(1-x^2)^{-\frac{1}{2}}.dx =$$

$$= x.\arcsin x + \frac{1}{2}\int -2x.(1-x^2)^{-\frac{1}{2}}.dx = x.\arcsin x + \frac{1}{2} \cdot \frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + k =$$

$$= x.\arcsin x + \sqrt{1-x^2} + k$$

66. $\int \sqrt{1-x^2}\,dx$ Sol: $\frac{1}{2}\left(\arcsin x + x\sqrt{1-x^2}\right) + k$

$$\int \sqrt{1-x^2}\,dx = \int \frac{1-x^2}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{-x^2}{\sqrt{1-x^2}}dx =$$

$$= \arcsin x + \int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx =$$

La integral que nos queda la realizaremos por partes:

$$\int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \int x \cdot \frac{-x}{\sqrt{1-x^2}} \cdot dx = \left\{\begin{matrix} u = x \Rightarrow dx \\ dv = \dfrac{-x}{\sqrt{1-x^2}}dx \Rightarrow v = \sqrt{1-x^2} \end{matrix}\right\} =$$

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,dx$$

Sustituyendo nos queda:

$$\int \sqrt{1-x^2}\,dx = \arcsin x + \int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \arcsin x + x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,dx$$

y se nos repite la misma integral. Entonces:

$$\int \sqrt{1-x^2}\,dx = \arcsin x + x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,dx \Rightarrow$$

$$\Rightarrow 2\int \sqrt{1-x^2}dx = \text{arc sen}\, x + x\sqrt{1-x^2} \Rightarrow \int \sqrt{1-x^2}dx = \frac{1}{2}\left(\text{arc sen}\, x + x\sqrt{1-x^2}\right) + k$$

67. $\int x\,\text{arc sen}\, x\, dx$ Sol: $\frac{1}{4}\left[(2x^2-1)\text{arcsen}\, x + x\sqrt{1-x^2}\right] + k$

Hacemos el siguiente cambio: $\left\{\begin{matrix} u = \text{arc sen}\, x \\ dv = xdx \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} du = \frac{1}{\sqrt{1-x^2}}\cdot dx \\ v = \frac{x^2}{2} \end{matrix}\right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\int x\text{arc sen}\, x\cdot dx = \frac{x^2}{2}\cdot \text{arc sen}\, x - \int \frac{x^2}{2}\cdot\frac{1}{\sqrt{1-x^2}}\cdot dx = \frac{x^2}{2}\cdot\text{arc sen}\, x + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}}dx =$$

Por el ejercicio anterior tenemos que :

$$\int \frac{-x^2}{\sqrt{1-x^2}}\cdot dx = \int x\cdot\frac{-x}{\sqrt{1-x^2}}\cdot dx = \left\{\begin{matrix} u = x \Rightarrow dx \\ dv = \frac{-x}{\sqrt{1-x^2}}dx \Rightarrow v = \sqrt{1-x^2} \end{matrix}\right\} =$$

$$= x\sqrt{1-x^2} - \int\sqrt{1-x^2}dx = x\sqrt{1-x^2} - \quad \frac{1-x^2}{\sqrt{1-x^2}}dx \quad \Rightarrow$$

$$\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \int\frac{1}{\sqrt{1-x^2}}dx - \int\frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \text{arc sen}\, x - \int\frac{-x^2}{\sqrt{1-x^2}}dx$$

En consecuencia:

$$\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \text{arc sen}\, x - \int\frac{-x^2}{\sqrt{1-x^2}}dx$$

Por tanto:

$$2\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \text{arc sen}\, x \Rightarrow \int\frac{-x^2}{\sqrt{1-x^2}}dx = \frac{1}{2}\left(x\sqrt{1-x^2} - \text{arc sen}\, x\right)$$

Sustituyendo obtenemos:

$$\int x\text{arc sen}\, x\cdot dx = \frac{x^2}{2}\cdot\text{arc sen}\, x + \frac{1}{2}\int\frac{-x^2}{\sqrt{1-x^2}}dx =$$

$$= \frac{x^2}{2} \cdot \text{arc sen}\, x + \frac{1}{2} \cdot \frac{1}{2}\left(x\sqrt{1-x^2} - \text{arc sen}\, x\right) + k =$$

$$= \frac{x^2}{2} \cdot \text{arc sen}\, x + \frac{1}{4} x\sqrt{1-x^2} - \frac{1}{4}\text{arc sen}\, x + k =$$

$$= \frac{1}{4}\left[(2x^2 - 1)\,\text{arcsen}\, x + x\sqrt{1-x^2}\right] + k$$

68. $\int \text{arc tg}\, x dx$ Sol: $x\,\text{arc tg}\, x - \frac{1}{2}\text{Ln}\,(1+x^2) + k$

$$\int \text{arc tg}\, x dx = \left\{\begin{array}{c} u = \text{arc tg}\, x \Rightarrow du = \frac{1}{1+x^2}dx \\ dv = dx \Rightarrow v = x \end{array}\right\} = x \cdot \text{arc tg}\, x - \int x \cdot \frac{1}{1+x^2}dx =$$

$$= x \cdot \text{arc tg}\, x - \int \frac{x}{1+x^2}dx = x \cdot \text{arc tg}\, x - \frac{1}{2}\int \frac{2x}{1+x^2}dx = x \cdot \text{arc tg}\, x - \frac{1}{2}\text{Ln}\,(1+x^2) + k$$

69. $\int \text{arc tg}\sqrt{x}\, dx$ Sol: $(x+1)\,\text{arc tg}\sqrt{x} - \sqrt{x} + k$

$$\int \text{arc tg}\sqrt{x}\, dx = \left\{\begin{array}{c} u = \text{arc tg}\sqrt{x} \Rightarrow du = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}dx \\ dv = dx \Rightarrow v = x \end{array}\right\} = x\text{arc tg}\sqrt{x} - \int x\frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}dx =$$

$$= x\text{arc tg}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{1+x} \cdot dx = \left\{\begin{array}{c} x = t^2 \Rightarrow \\ \Rightarrow dx = 2tdt \end{array}\right\} = x\text{arc tg}\sqrt{x} - \frac{1}{2}\int \frac{t}{1+t^2} \cdot 2tdt =$$

$$= x\text{arc tg}\sqrt{x} - \int \frac{t^2}{1+t^2}dt = x\text{arc tg}\sqrt{x} - \int \frac{1+t^2-1}{1+t^2}dt =$$

$$= x\text{arc tg}\sqrt{x} - \int \frac{1+t^2-1}{1+t^2}dt = x\text{arc tg}\sqrt{x} - \int \left(1 - \frac{1}{1+t^2}\right)dt =$$

$$= x\text{arc tg}\sqrt{x} - \int dt + \int \frac{1}{1+t^2}dt = x\text{arc tg}\sqrt{x} - t + \text{arc tg}\, t + k =$$

$$= x\text{arc tg}\sqrt{x} - \sqrt{x} + \text{arc tg}\sqrt{x} + k = (x+1)\,\text{arc tg}\sqrt{x} - \sqrt{x} + k$$

70. $\int \mathbf{Ln}\,(x+\sqrt{1+x^2})dx$ Sol: $x\,\mathbf{Ln}\,(x+\sqrt{1+x^2})-\sqrt{1+x^2}+k$

Hacemos: $u=\mathbf{Ln}\,(x+\sqrt{1+x^2})$ y $dv=dx$ con lo cual

$$du=\frac{1}{x+\sqrt{1+x^2}}\cdot\left(1+\frac{2x}{2\sqrt{1+x^2}}\right)dx=\frac{1}{x+\sqrt{1+x^2}}\cdot\left(1+\frac{x}{\sqrt{1+x^2}}\right)dx\Rightarrow$$

$$\Rightarrow du=\frac{1}{x+\sqrt{1+x^2}}\cdot\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right)dx=\frac{1}{\sqrt{1+x^2}}dx \quad \text{y} \quad v=x$$

Sustituyendo en la fórmula de integración por partes, obtenemos:

$$\int \mathbf{Ln}\,(x+\sqrt{1+x^2})dx=x\cdot\mathbf{Ln}\,(x+\sqrt{1+x^2})-\int x\cdot\frac{1}{\sqrt{1+x^2}}dx=$$

$$=x\cdot\mathbf{Ln}\,(x+\sqrt{1+x^2})-\int\frac{2x}{2\sqrt{1+x^2}}dx=x\cdot\mathbf{Ln}\,(x+\sqrt{1+x^2})-\sqrt{1+x^2}+k$$

71. $\int \frac{x\,\mathbf{arc\,sen}\,x}{\sqrt{1-x^2}}dx$ Sol: $x-\sqrt{1-x^2}\,\mathbf{arc\,sen}\,x+k$

$$\int \frac{x\,\mathbf{arc\,sen}\,x}{\sqrt{1-x^2}}dx=\left\{\begin{array}{c} u=\mathbf{arc\,sen}\,x\Rightarrow du=\frac{1}{\sqrt{1-x^2}}dx \\ dv=\frac{x}{\sqrt{1-x^2}}dx\Rightarrow v=-\int\frac{-2x}{2\sqrt{1-x^2}}dx=-\sqrt{1-x^2}\end{array}\right\}=$$

$$=-\sqrt{1-x^2}\cdot\mathbf{arc\,sen}\,x-\int -\sqrt{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}}dx=-\sqrt{1-x^2}\cdot\mathbf{arc\,sen}\,x+\int dx=$$

$$=-\sqrt{1-x^2}\cdot\mathbf{arc\,sen}\,x+x+k$$

72. $\displaystyle\int \frac{2x-1}{(x-1)(x-2)}dx$ Sol: $\text{Ln}\left|\frac{(x-2)^3}{x-1}\right|+k$

Tenemos una integral de tipo racional donde el grado del numerador es menor que el grado del denominador. Vamos a descomponer el integrando en fracciones simples:

$$(x-1)(x-2)=0 \Rightarrow \begin{cases} x=1 \\ x=2 \end{cases} \text{(raíces reales simples)}$$

Entonces:

$$\frac{2x-1}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2}=\frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

Vamos a calcular los coeficientes indeterminados. Al ser los denominadores iguales, los numeradores también lo serán. Por tanto:

$$2x-1=A(x-2)+B(x-1) \Rightarrow \begin{cases} x=1 \to 1=-A \Rightarrow A=-1 \\ x=2 \to 3=B \quad \Rightarrow B=3 \end{cases}$$

Por tanto,

$$\int \frac{2x-1}{(x-1)(x-2)}dx=\int\left(\frac{-1}{x-1}+\frac{3}{x-2}\right)dx=-\int \frac{1}{x-1}dx+3\int \frac{1}{x-2}dx=$$

$$=-\text{Ln }(x-1)+3 \quad \text{Ln }(x-2)+k=\text{Ln}\left|\frac{(x-2)^3}{x-1}\right|+k$$

73. $\displaystyle\int \frac{xdx}{(x+1)(x+3)(x+5)}$ Sol: $\frac{1}{8}\text{ Ln}\left|\frac{(x+3)^6}{(x+1)(x+5)^5}\right|+k$

Tenemos una integral de tipo racional donde el grado del numerador es menor que el grado del denominador. Vamos a descomponer el integrando en fracciones simples:

$$(x+1)(x+3)(x+5)=0 \Rightarrow \begin{cases} x=-1 \\ x=-3 \\ x=-5 \end{cases} \quad \text{(raíces reales simples)}$$

Entonces:

$$\frac{x}{(x+1)(x+3)(x+5}=\frac{A}{x+1}+\frac{B}{x+3}+\frac{C}{x+5}=$$

$$=\frac{A(x+3)(x+5)+B(x+1)(x+5)+C(x+1)(x+3)}{(x+1)(x+3)(x+5)}$$

Para calcular los coeficientes indeterminados, al ser los denominadores iguales, los numeradores también lo serán. Por tanto:

$$\boldsymbol{x = A(x+3)(x+5) + B(x+1)(x+5) + C(x+1)(x+3) \Rightarrow \begin{cases} x=-1 \rightarrow -1 = 8A \Rightarrow A = -\frac{1}{8} \\ x=-3 \rightarrow -3 = -4B \ \Rightarrow B = \frac{3}{4} \\ x=-5 \rightarrow -5 = 8C \Rightarrow C = -\frac{5}{8} \end{cases}}$$

Por tanto,

$$\boldsymbol{\int \frac{xdx}{(x+1)(x+3)(x+5)} = \int \left(\frac{-\frac{1}{8}}{x+1} + \frac{\frac{3}{4}}{x+3} + \frac{-\frac{5}{8}}{x+5} \right) dx =}$$

$$\boldsymbol{= -\frac{1}{8}\int \frac{1}{x+1}dx + \frac{3}{4}\int \frac{1}{x+3}dx - \frac{5}{8}\int \frac{1}{x+5}dx = -\frac{1}{8}\mathrm{Ln}(x+1) + \frac{3}{4}\mathrm{Ln}(x+3) - \frac{5}{8}\mathrm{Ln}(x+5) + k =}$$

$$\boldsymbol{= \frac{1}{8}\left(\mathrm{Ln}\ (x+1) + 6\,\mathrm{Ln}\ (x+3) - 5\,\mathrm{Ln}\ (x+5)\right) + k = \frac{1}{8}\mathrm{Ln}\left| \frac{(x+3)^6}{(x+1)(x+5)^5} \right| + k}$$

74. $\boldsymbol{\int \frac{x^5 + x^4 - 8}{x^3 - 4x} dx}$ Sol: $\frac{x^3}{3} + \frac{x^2}{2} + 4x + \mathrm{Ln}\left| \frac{x^2(x-2)^5}{(x+2)^3} \right| +$

Al ser el grado del numerador mayor que el grado del denominador, antes de aplicar el método de descomposición en fracciones simples tendremos que dividir. De esta forma obtenemos:

$$\frac{x^5 + x^4 - 8}{x^3 - 4x} = x^2 + x + 4 + \frac{4x^2 + 16x - 8}{x \quad\ x}$$

En consecuencia:

$$\int \frac{x^5 + x^4 - 8}{x^3 - 4x} \cdot dx = \int (x^2 + x + 4)dx + \int \frac{4x^2 + 16x - 8}{x \quad\ x} \cdot dx =$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \frac{4x^2 + 16x \quad 8}{x^3 - 4} \cdot dx$$

A la integral que nos queda le aplicamos el método de descomposición en fracciones simples. Calculamos las raíces del denominador:

$$x^3-4x=0 \quad \rightarrow \quad x\cdot(x \quad -4)=0 \quad \rightarrow \quad \begin{cases} x=0 \\ x=\pm 2 \end{cases}$$

Entonces:

$$\frac{4x^2+16x-8}{x^3-4x}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+2}=\frac{A(x-2)(x+2)+Bx(x+2)+Cx(x-2)}{x(x-2)(x+2)}$$

Como los denominadores son iguales, los numeradores también lo serán; por tanto:

$$4x^2+16x-8=A(x-2)(x+2)+Bx(x+2)+Cx(x-2)$$

Calculamos los coeficientes indeterminados: le vamos asignando los valores de las raíces

$$\begin{array}{lllll} x=0 & \rightarrow & -8=-4A & \rightarrow & A=2 \\ x=2 & \rightarrow & 40=\ \ 8B & \rightarrow & B=5 \\ x=-2 & \rightarrow & -24=8C & \rightarrow & C=-3 \end{array}$$

Por tanto, la fracción descompuesta en fracciones simples nos queda:

$$\frac{4x^2+16 \quad 8}{x^3-4x}=\frac{2}{x}+\frac{5}{x-2}-\frac{3}{x+2}$$

La integral de la función pedida será:

$$\int \frac{x^5+x^4-8}{x^3-4x}\cdot dx=\frac{x^3}{3}+\frac{x^2}{2}+4x+ \quad \frac{4x^2+16x-8}{x \quad 4x}\cdot dx=$$

$$=\frac{x^3}{3}+\frac{}{2}+4x+\int \quad \frac{2}{x}+\frac{5}{x-2}-\frac{3}{x \quad 2}\Big)\cdot dx$$

$$=\frac{x^3}{3}+\frac{}{2}+4x+\int \frac{2}{x}\cdot dx+\int \frac{5}{x-2}\cdot dx-\int \frac{3}{x+2}\cdot dx=$$

$$=\frac{x^3}{3}+\frac{}{2}+4x+2\int \frac{1}{x}\cdot dx+5\int \frac{1}{x-2}\cdot dx-3\int \frac{1}{x \quad 2}\cdot dx=$$

$$=\frac{x^3}{3}+\frac{}{}+4x+2\cdot \text{Ln}\,|\,x\,|+5\cdot \text{Ln}\,|\,x-2\,|-3\cdot \text{Ln}\,|\,x+2\,|+k=$$

$$=\frac{x^3}{3}+\frac{x^2}{2}+4x+\text{Ln}\left|\frac{x^2(x-2)^5}{(x+2)^3}\right|+$$

75. $\int \frac{x^4 dx}{(x^2-1)(x+2)}$ Sol: $\frac{x^2}{2} - 2x + \frac{1}{6}\ \text{Ln}\left|\frac{(\ \ -1)}{(x+1)^3}\right| + \frac{16}{3}\cdot \text{Ln}\,|x+2| + k$

Como el grado del numerador es mayor que el del denominador, tenemos que dividir, obteniendo:

$$\frac{x^4}{(x^2-1)(x+2)} = x - 2 + \frac{5\ \ -}{(x\ \ -1)(x+2)}$$

Con lo que

$$\int \frac{x^4 \cdot dx}{(x^2-1)(x+2)} = \int (x-2)\,dx + \int \frac{5x^2-4}{(x^2-1)(x+2)}\,dx = \frac{x^2}{2} - 2x + \int \frac{5x^2-4}{(x^2-1)(x+2)}\,dx$$

y tendremos que integrar la función racional que nos queda, donde el grado del numerador es menor que el grado del denominador.

Descomponemos en fracciones simples:

$$\frac{5x^2-4}{(x^2-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2} = \frac{A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1)}{(x\ \ -1)(x+2)}$$

Como los denominadores son iguales, también lo serán los numeradores. Entonces:

$$5x^2 - 4 = A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1)$$

Calculamos los coeficientes indeterminados:

$$x = 1 \quad \to \quad 1 = 6A \quad \to \quad A = \frac{1}{6}$$

$$x = -1 \quad \to \quad 1 = -2B \quad \to \quad B = -\frac{1}{2}$$

$$x = -2 \quad \to \quad 16 = 3C \quad \to \quad C = \frac{16}{3}$$

Entonces: $$\frac{5x^2 -}{(x^2-1)(x+2)} = \frac{\frac{1}{6}}{x-1} + \frac{-\frac{1}{2}}{x+1} + \frac{\frac{16}{\ }}{x+2}$$

Y, por tanto:

$$\int \frac{x^4 \;\; dx}{(x^2-1)(x+2)} = \frac{x^2}{2} - 2x + \int \frac{5x^2 -}{(x \;\; -1)(x+2)} dx = \frac{x^2}{2} - \;\; x + \int \left(\frac{\frac{1}{6}}{x-1} + \frac{-\frac{1}{2}}{x+1} + \frac{\frac{16}{\;}}{x+2} \right) \cdot dx =$$

$$= \frac{x^2}{2} - 2x + \int \frac{\frac{1}{6}}{x-1} \cdot dx + \int \frac{-\frac{1}{2}}{x+1} \cdot dx + \int \frac{\frac{16}{\;}}{x+2} \cdot dx =$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} \int \frac{1}{x-1} \cdot dx - \frac{1}{2} \int \frac{1}{x+1} \cdot dx + \frac{16}{3} \int \frac{1}{x+2} \cdot dx =$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot \text{Ln}\,|x-1| - \frac{1}{2} \cdot \text{Ln}\,|x+1| + \frac{16}{3} \cdot \text{Ln}\,|x+2| + k =$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot \left(\text{Ln}\,|x-1| - 3 \cdot \text{Ln}\,|x+1| \right) + \frac{16}{3} \cdot \text{Ln}\,|x+2| + k =$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot \text{Ln} \left| \frac{-1}{(x+1)^3} \right| + \frac{16}{3} \cdot \text{Ln}\,|x+2| + k$$

76. $\int \frac{dx}{(x-1)^2 (x-2)}$ Sol: $\frac{1}{x-1} + \text{Ln} \left| \frac{x-}{\;} \right| \;\; k$

Como el grado del numerador es menor que el grado del denominador aplicamos la descomposición en fracciones simples directamente:

$$\frac{1}{(x-1)^2 (x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-2)} = \frac{A(x-1)(x-2) + B(x-2) + C(x-1)^2}{(x-1)^2 (x-2)} \quad \rightarrow$$

$$\rightarrow \quad 1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

Calculamos los coeficientes:

$x = 1: \quad 1 = -B \quad \rightarrow \quad B = -1$

$x = 2: \quad 1$

$x = 0: \quad 1 = 2A - 2B + C \quad \rightarrow \quad 1 = 2A + 2 + 1 \quad \rightarrow \quad A = -1$

Entonces:

$$\int \frac{1}{(x-1) \;\; (x-2)} \cdot dx = \int \frac{-1}{x-1} \cdot dx + \int \frac{1}{(x-1)^2} \cdot dx + \int \frac{1}{(x-2)} \cdot dx =$$

$$= -\int \frac{1}{x-1}\cdot dx - \int (x-1)^{-2}\cdot dx + \int \frac{1}{x-2}\cdot dx =$$

$$= -\text{Ln}\,|x-1| - \frac{(x-1)^{-1}}{-1} + \text{Ln}\,|x-2| + k = \frac{1}{x-1} + \text{Ln}\left|\frac{x-2}{x-1}\right| + k$$

77. $\displaystyle\int \frac{x-8}{x^3-4x^2+4x}dx$ Sol: $\displaystyle\frac{3}{x-2} + \text{Ln}\,\frac{(x-2)^2}{x^2} + k$

Igual que en el anterior, aplicamos la descomposición en fracciones simples:

Calculamos las raíces del denominador:

$$x^3-4x^2+4x=0 \;\rightarrow\; x\cdot(x^2-4x+4)=0 \;\rightarrow\; x\cdot(x-2)^2=0 \;\rightarrow\; \begin{cases} x=0 \\ x=2 \quad \text{(doble)} \end{cases}$$

Entonces:

$$\frac{x-8}{x^3-4x^2+4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} = \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2} \quad \Rightarrow$$

$$\Rightarrow \quad x-8 = A(x-2)^2 + Bx(x-2) + Cx \quad \Rightarrow$$

Calculamos los coeficientes:

$$x=0 \;\rightarrow\; -8=4A \;\rightarrow\; A=-2$$
$$x=2 \;\rightarrow\; -6=2C \;\rightarrow\; C=-3$$
$$x=1 \;\rightarrow\; -7=A-B+C \;\rightarrow\; B=7+A+C=7-2-3=2 \;\rightarrow\; B=2$$

Entonces:

$$\int \frac{x-8}{x^3-4x^2+4x}\,dx = \int\left(\frac{-2}{x} + \frac{2}{x-2} + \frac{-3}{(x-2)^2}\right)\cdot dx =$$

$$= -2\int \frac{1}{x}\cdot dx + 2\int \frac{1}{x-2}\cdot dx - 3\int \frac{1}{(x-2)^2}\cdot dx = -2\,\text{Ln}\,|x| + 2\,\text{Ln}\,|x-2| - 3\int (x-2)^{-2}dx =$$

$$= -2\,\text{Ln}\,|x| + 2\,\text{Ln}\,|x-2| - 3\cdot\frac{(x-2)^{-1}}{-1} + k = \frac{3}{x-2} + \text{Ln}\,\frac{(x-2)^2}{x^2} + k$$