Integrales inmediatas de tipo potencial

- $\int x^2(3x^3+14)^3\,dx = \frac{1}{9}\int 9x^2(3x^3+14)^3\,dx = \frac{1}{9}\frac{1}{4}\int 4\cdot 9x^2(3x^3+14)^3\,dx =$

  $= \frac{1}{36}(3x^3+14)^4 + C$

- $\int \sqrt[5]{5x+6}\,dx = \int (5x+6)^{1/5}\,dx = \frac{1}{5}\int 5(5x+6)^{1/5}\,dx =$

  $= \frac{1}{5}\frac{(5x+6)^{1/5+1}}{\frac{1}{5}+1} = \frac{1}{6}\sqrt[5]{(5x+6)^6} + C$

- $\int \frac{17x}{\sqrt[3]{6x^2+8}}\,dx = 17\int x(6x^2+8)^{-1/3}\,dx = 17\frac{1}{12}\int 12x(6x^2+8)^{-1/3}\,dx =$

  $= \frac{17}{12}\frac{(6x^2+8)^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} = \frac{51}{24}\sqrt[3]{(6x^2+8)^2} + C$

- $\int \frac{\operatorname{arctg} x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\operatorname{arctg} x\,dx = \frac{1}{2}\int 2\frac{1}{1+x^2}\operatorname{arctg} x\,dx = \frac{1}{2}(\operatorname{arctg} x)^2 + C$

- $\int \sec^2 x\sqrt{\operatorname{tg} x}\,dx = \int \frac{1}{\cos^2 x}(\operatorname{tg} x)^{1/2}\,dx = \frac{(\operatorname{tg} x)^{1/2+1}}{\frac{1}{2}+1} = \frac{2}{3}(\operatorname{tg} x)^{3/2} + C$

- $\int \frac{dx}{(3x+1)^4} = \int (3x+1)^{-4}\,dx = \frac{1}{3}\int 3(3x+1)^{-4}\,dx = \frac{1}{3}\frac{(3x+1)^{-4+1}}{-4+1} =$

  $= -\frac{1}{9}(3x+1)^{-3} + C$

- $\int \cos x\cdot \operatorname{sen}^3 x\,dx = \frac{1}{4}\int 4\cos x\,\operatorname{sen}^3 x\,dx = \frac{1}{4}\operatorname{sen}^4 x + C$

- $\int \frac{(x+3)}{(x^2+6x)^{1/3}}\,dx = \int (x+3)(x^2+6x)^{-1/3}\,dx = \frac{1}{2}\int 2(2x+6)(x^2+6x)^{-1/3}\,dx =$

  $= \frac{1}{2}\int (2x+6)(x^2+6x)^{-1/3}\,dx = \frac{1}{2}\frac{(x^2+6x)^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} = \frac{3}{4}(x^2+6x)^{2/3} + C$

- $\int \sqrt{x^2-2x^4}\,dx = \int \sqrt{x^2(1-2x^2)}\,dx = \int x\sqrt{1-2x^2} = \int x(1-2x^2)^{1/2}\,dx =$

  $= -\frac{1}{4}\int -4x(1-2x^2)^{1/2}\,dx = -\frac{1}{4}\frac{(1-2x^2)^{1/2+1}}{\frac{1}{2}+1} = -\frac{1}{6}(1-2x^2)^{3/2} + C$

Integrales inmediatas de tipo exponencial

- $\int x^2 \cdot 7^{x^3+5}\,dx = \frac{1}{3}\int 3x^2 \cdot 7^{x^3+5}\,dx = \frac{1}{3\ln 7}\,7^{x^3+5} + C$

- $\int \frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}}\,5^{\sqrt{x}}\,dx = 2\int \frac{1}{2\sqrt{x}}\,5^{\sqrt{x}}\,dx = \frac{2}{\ln 5}\int \frac{1}{2\sqrt{x}}\,5^{\sqrt{x}}\ln 5\,dx =$

  $= \frac{2}{\ln 5}\,5^{\sqrt{x}} + C$

- $\int \frac{e^{\operatorname{arc\,sen} x}}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}}\,e^{\operatorname{arcsen} x}\,dx = e^{\operatorname{arc\,sen} x} + C$

- $\int \cos 3x \cdot e^{\operatorname{sen} 3x}\,dx = \frac{1}{3}\int 3\cos 3x \cdot e^{\operatorname{sen} 3x}\,dx = \frac{1}{3}e^{\operatorname{sen} 3x} + C$

- $\int \frac{6^{\ln x}}{x}\,dx = \int \frac{1}{x}\,6^{\ln x}\,dx = \frac{1}{\ln 6}\int \frac{1}{x}\,6^{\ln x}\ln 6\,dx = \frac{1}{\ln 6}\,6^{\ln x} + C$

Integrales inmediatas de tipo logarítmico

- $\int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}\,dx = -\int -\frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}\,dx = -\int \frac{\cos x - \operatorname{sen} x}{\operatorname{sen} x + \cos x}\,dx =$

  $= -\ln|\operatorname{sen} x + \cos x| + C$

- $\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln|x^2+1| + C$

- $\int \frac{27x^2+30x+3}{3x^3+5x^2+x-1}\,dx = \int \frac{3(9x^2+10x+1)}{3x^3+5x^2+x-1}\,dx = 3\int \frac{9x^2+10x+1}{3x^3+5x^2+x-1}\,dx =$

  $= 3\ln|3x^3+5x^2+x-1| + C$

- $\int \frac{1}{\operatorname{tg} x}\,dx = \int \frac{1}{\frac{\operatorname{sen} x}{\cos x}}\,dx = \int \frac{\cos x}{\operatorname{sen} x}\,dx = \ln|\operatorname{sen} x| + C$

- $\int \frac{7^{2x}}{7^{2x}+5}\,dx = \frac{1}{2}\int \frac{2\cdot 7^{2x}}{7^{2x}+5}\,dx = \frac{1}{2}\,\frac{1}{\ln 7}\int \frac{2\cdot 7^{2x}\cdot \ln 7}{7^{2x}+5}\,dx =$

  $= \frac{1}{2\ln 7}\ln|7^{2x}+5| + C$

- $\int \frac{1}{x \ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln|\ln x| + C$

- $\int \frac{1}{\cos^2 x \cdot tg\, x} dx = \int \frac{\frac{1}{\cos^2 x}}{tg\, x} dx = \ln|tg\, x| + C$

- $\int \frac{1}{\sqrt{x}(1-\sqrt{x})} dx = \int \frac{\frac{1}{\sqrt{x}}}{1-\sqrt{x}} dx = -2 \int \frac{\frac{1}{-2\sqrt{x}}}{1-\sqrt{x}} dx = -2\ln|1-\sqrt{x}| + C$

Integración por partes

- $\int arctg\, x\, dx = \left[\begin{array}{l} u = arctg\, x \longrightarrow du = \frac{1}{1+x^2} dx \\ dv = dx \longrightarrow v = x \end{array} \right] = x \cdot arctg\, x -$

$- \int x \frac{1}{1+x^2} dx = x\, arctg\, x - \frac{1}{2} \int \frac{2x}{1+x^2} dx = x\, arctg\, x - \frac{1}{2} \ln|1+x^2| + C$

- $\int arcsen\, x\, dx = \left[\begin{array}{l} u = arcsen\, x \longrightarrow du = \frac{1}{\sqrt{1-x^2}} dx \\ dv = dx \longrightarrow v = x \end{array} \right] =$

$= x\, arcsen\, x - \int \frac{x}{\sqrt{1-x^2}} dx = x\, arcsen\, x - \underbrace{\frac{1}{2} 2 \int \frac{2x\, (1-x^2)^{-1/2}}{2} dx}_{\text{inmediata de tipo potencial}} =$

$= x\, arcsen\, x - (1-x^2)^{1/2} = x\, arcsen\, x - \sqrt{1-x^2} + C$

- $\int \ln(x+1)\, dx = \left[\begin{array}{l} u = \ln(x+1) \longrightarrow du = \frac{1}{x+1} dx \\ dv = dx \longrightarrow v = x \end{array} \right] =$

$= x \ln|x+1| - \int x \frac{1}{x+1} dx = x \ln|x+1| - x + \ln|x+1| + C$

Calculamos $\int \frac{x}{x+1} dx = \int 1\, dx - \int \frac{1}{x+1} dx = x - \ln|x+1|$

$$\begin{array}{r|l} x & x+1 \\ \hline -x-1 & 1 \\ \hline -1 & \end{array}$$

de donde $\frac{x}{x+1} = 1 - \frac{1}{x+1}$

- $\int x\, arctg\, x\, dx = \left[\begin{array}{l} u = arctg\, x \longrightarrow du = \frac{1}{1+x^2} dx \\ dv = x\, dx \longrightarrow v = \frac{x^2}{2} \end{array} \right] =$

$= \frac{x^2}{2} arctg\, x - \int \frac{x^2}{2} \frac{1}{1+x^2} dx = \frac{x^2}{2} arctg\, x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \underset{(1)}{=}$

Calculamos $\displaystyle\int \frac{x^2}{1+x^2}\,dx = \int 1\,dx - \int \frac{1}{1+x^2}\,dx = x - \operatorname{arctg} x$

$$\begin{array}{r|l} x^2 & x^2+1 \\ \cline{2-2} -x^2-1 & 1 \\ \hline -1 & \end{array} \qquad \text{de donde } \frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$$

$$\underset{①}{=} \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\left(x - \operatorname{arctg} x\right) + C$$

- $\displaystyle\int x \ln(x+1)\,dx = \left[\begin{array}{l} u = \ln(x+1) \longrightarrow du = \frac{1}{x+1}\,dx \\ dv = x\,dx \longrightarrow v = \frac{x^2}{2} \end{array}\right] =$

$$= \frac{x^2}{2}\ln|x+1| - \int \frac{x^2}{2}\frac{1}{x+1}\,dx = \frac{x^2}{2}\ln(x+1) - \frac{1}{2}\int \frac{x^2}{x+1}\,dx \underset{①}{=}$$

Calculamos $\displaystyle\int \frac{x^2}{1+x}\,dx = \int (x-1)\,dx + \int \frac{1}{x+1}\,dx = \frac{x^2}{2} - x + \ln|x+1|$

$$\begin{array}{r|l} x^2 & x+1 \\ \cline{2-2} -x^2-x & x-1 \\ \hline -x & \\ x+1 & \\ \hline 1 & \end{array} \qquad \text{de donde } \frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$$

$$\underset{①}{=} \frac{x^2}{2}\ln|x+1| - \frac{1}{2}\left(\frac{x^2}{2} - x + \ln|x+1|\right) + C$$

- $\displaystyle\int (3x^2+2x-7)\cos x\,dx = \left[\begin{array}{l} u = 3x^2+2x-7 \longrightarrow du = (6x+2)\,dx \\ dv = \cos x\,dx \longrightarrow v = \operatorname{sen} x \end{array}\right] =$

$$= (3x^2+2x-7)\operatorname{sen} x - \underbrace{\int (6x+2)\operatorname{sen} x\,dx}_{\text{por partes}} \underset{①}{=}$$

Calculamos $\displaystyle\int (6x+2)\operatorname{sen} x\,dx = \left[\begin{array}{l} u = 6x+2 \longrightarrow du = 6\,dx \\ dv = \operatorname{sen} x\,dx \longrightarrow v = -\cos x \end{array}\right] =$

$$= -(6x+2)\cos x + 6\int \cos x\,dx = -(6x+2)\cos x + 6\operatorname{sen} x$$

$$\underset{①}{=} (3x^2+2x-7)\operatorname{sen} x + (6x+2)\cos x - 6\operatorname{sen} x + C$$

- $\displaystyle\int (5x^2-3)\,4^{3x+1}\,dx = \left[\begin{array}{l} u = 5x^2-3 \longrightarrow du = 10x\,dx \\ dv = 4^{3x+1}\,dx \longrightarrow v = \underbrace{\int 4^{3x+1}\,dx}_{\substack{\text{inmediata de} \\ \text{tipo exponencial}}} = \frac{1}{4\ln 4}4^{3x+1} \end{array}\right] =$

$$= (5x^2-3)\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}\underbrace{\int 10x \cdot 4^{3x+1}\,dx}_{\text{por partes}} \underset{①}{=}$$

Calculamos $\int 10x \cdot 4^{3x+1}\,dx = \left[\begin{array}{l} u = 10x \longrightarrow du = 10\,dx \\ dv = 4^{3x+1}\,dx \rightarrow v = \frac{1}{3\ln 4}4^{3x+1} \end{array}\right] =$

$$= 10x\,\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}10\int 4^{3x+1}\,dx =$$

$$= 10x\,\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}10\,\frac{1}{3\ln 4}4^{3x+1}$$

$$\underset{1}{=} (5x^2-3)\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}\left(10x\,\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}10\,\frac{4^{3x+1}}{3\ln 4}\right) + C$$

- $\int \frac{x}{\operatorname{sen}^2 x}\,dx = \left[\begin{array}{l} u = x \longrightarrow du = dx \\ dv = \frac{1}{\operatorname{sen}^2 x}\,dx \longrightarrow v = \int \frac{1}{\operatorname{sen}^2 x}\,dx = -\operatorname{cotg} x \end{array}\right] =$

$$= -x\operatorname{cotg} x + \int \operatorname{cotg} x\,dx = -x\operatorname{cotg} x + \int \frac{\cos x}{\operatorname{sen} x}\,dx =$$

$$= -x\operatorname{cotg} x + \ln|\operatorname{sen} x| + C$$

- $\int \sec^4 x\,dx = \int \frac{1}{\cos^4 x}\,dx = \int \frac{1}{\cos^2 x \cdot \cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\cdot\frac{1}{\cos^2 x}\,dx =$

$$= \left[\begin{array}{l} u = \frac{1}{\cos^2 x} \longrightarrow du = \frac{2\operatorname{sen} x}{\cos^3 x}\,dx \\ dv = \frac{1}{\cos^2 x}\,dx \longrightarrow v = \operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x} \end{array}\right] = \frac{\operatorname{sen} x}{\cos^3 x} - 2\int \frac{\operatorname{sen}^2 x}{\cos^4 x}\,dx =$$

$$= \frac{\operatorname{sen} x}{\cos^3 x} - 2\int \frac{1-\cos^2 x}{\cos^4 x}\,dx = \frac{\operatorname{sen} x}{\cos^3 x} - 2\int \frac{1}{\cos^4 x}\,dx + 2\int \frac{1}{\cos^2 x}\,dx =$$

$$= \frac{\operatorname{sen} x}{\cos^3 x} - 2\underbrace{\int \frac{1}{\cos^4 x}\,dx}_{\text{Es la integral que estamos calculando}} + 2\operatorname{tg} x \Rightarrow \text{(pasando } -2\int \frac{1}{\cos^4 x}\,dx \text{ a la izqda.)}$$

$$3\int \frac{1}{\cos^4 x}\,dx = \frac{\operatorname{sen} x}{\cos^3 x} + 2\operatorname{tg} x \Rightarrow \int \frac{1}{\cos^4 x}\,dx = \frac{1}{3}\left(\frac{\operatorname{sen} x}{\cos^3 x} + 2\operatorname{tg} x\right) + C$$

- $\int \frac{\ln x}{\sqrt{x}}\,dx = \left[\begin{array}{l} u = \ln x \longrightarrow du = \frac{1}{x}\,dx \\ dv = \frac{1}{\sqrt{x}}\,dx \longrightarrow v = \int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} \end{array}\right] =$

$$= 2\sqrt{x}\ln x - 2\int \frac{\sqrt{x}}{x}\,dx = 2\sqrt{x}\ln x - 4\sqrt{x} + C$$

Integración por cambio de variable

- $$\int \frac{3x}{\sqrt{1+7x^2}}dx = \left[\begin{matrix} t=\sqrt{1+7x^2} \rightarrow t^2 = 1+7x^2 \\ dx = \frac{t}{7x}\,dt \end{matrix}\right] = \int \frac{3x}{\sqrt{t^2}}\,\frac{t}{7x}\,dt =$$

$$= \frac{3}{7}\int dt = \frac{3t}{7} = \frac{3}{7}\sqrt{1+7x^2} + C$$

También se puede resolver como una integral inmediata

- $$\int \frac{5}{x\ln x}dx = \left[\begin{matrix} t=\ln x \rightarrow dt = \frac{1}{x}dx \\ dx = x\,dt \end{matrix}\right] = \int \frac{5}{x\cdot t}\,x\,dt =$$

$$= 5\int \frac{1}{t}dt = 5\ln|t| = 5\ln|\ln x| + C$$

- $$\int \frac{5}{\sqrt{9-4x^2}}dx = \underbrace{\left[\begin{matrix} 2x = 3\,\text{sen}\,t \rightarrow x = \frac{3}{2}\text{sen}\,t \\ dx = \frac{3}{2}\cos t\,dt \end{matrix}\right]}_{\text{En este caso el cambio de variable nos lo dan}} = \int \frac{5}{\sqrt{9-9\,\text{sen}^2 t}}\,\frac{3}{2}\cos t\,dt =$$

$$= \frac{15}{2}\int \frac{\cos t}{\sqrt{9}\sqrt{1-\text{sen}^2 t}}dt = \frac{5}{2}\int \frac{\cos t}{\cos t}dt = \frac{5}{2}\int dt = \frac{5}{2}t =$$

$\boxed{\text{sen}^2 t + \cos^2 t = 1}$ Hay que sabérsela

$$= \frac{5}{2}\,\text{arc sen}\left(\frac{2x}{3}\right) + C$$

ya que si $x = \frac{3}{2}\text{sen}\,t \rightarrow \frac{2x}{3} = \text{sen}\,t \rightarrow \text{arcsen}\left(\frac{2x}{3}\right) = \text{arc sen}(\text{sen}\,t) \rightarrow$

$\rightarrow \text{arcsen}\left(\frac{2x}{3}\right) = t$

- $$\int \sqrt{4-x^2}\,dx = \left[\begin{matrix} x = 2\,\text{sen}\,t \rightarrow dx = 2\cos t\,dt \\ \text{El cambio de variable nos lo dan} \end{matrix}\right] = \int \sqrt{4-4\,\text{sen}^2 t}\cdot 2\cos t\,dt =$$

$$= \int 2\cos t \cdot 2\cos t\,dt = 4\int \cos^2 t\,dt = 4\int \frac{1+\cos 2t}{2}dt =$$

$\boxed{\cos^2 t = \frac{1+\cos 2t}{2}}$ Hay que sabérsela

$$= 2\int (1+\cos 2t)\,dt = 2\left(t + \frac{\text{sen}\,2t}{2}\right) = 2t + \text{sen}\,2t + C =$$

$$= 2\,\text{arcsen}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{2} + C$$

ya que si $x = 2\,\text{sen}\,t \rightarrow \text{sen}\,t = \frac{x}{2} \rightarrow t = \text{arc sen}\left(\frac{x}{2}\right)$

y $\underbrace{\boxed{\text{sen}(2t) = 2\,\text{sen}\,t\cos t}}_{\text{Hay que sabérsela}} = 2\,\text{sen}\,t\cdot\sqrt{1-\text{sen}^2 t} = x\sqrt{1-\left(\frac{x}{2}\right)^2} = \frac{x\sqrt{4-x^2}}{2}$

- $\int \frac{1}{3+2x}\,dx = \left[\begin{matrix} t=3+2x \longrightarrow & dt = 2\,dx \\ & dx = \frac{1}{2}\,dt \end{matrix}\right] = \int \frac{1}{t}\,\frac{1}{2}\,dt =$

$= \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| = \frac{1}{2}\ln|3+2x| + C$

También se puede resolver como una integral inmediata

- $\int \frac{x}{2-x^2}\,dx = \left[\begin{matrix} t=2-x^2 \longrightarrow & dt = -2x\,dx \\ & dx = \frac{-1}{2x}\,dt \end{matrix}\right] = \int \frac{1}{t}\cdot\frac{-1}{2x}\,x\,dt =$

$= -\frac{1}{2}\int \frac{1}{t}\,dt = -\frac{1}{2}\ln|t| = -\frac{1}{2}\ln|2-x^2| + C$

- $\int e^{\operatorname{sen} x}\cdot\cos x\,dx = \left[t = \operatorname{sen} x \rightarrow dt = \underbrace{\cos x\,dx}_{\text{es lo que tenemos en la integral}} \right] = \int e^t\,dt =$

$= e^t = e^{\operatorname{sen} x} + C$

- $\int \frac{x}{1+x^4}\,dx = \left[\begin{matrix} t=x^2 \longrightarrow & dt = 2x\,dx \\ & \frac{dt}{2} = x\,dx \end{matrix}\right] = \int \frac{1}{1+t^2}\cdot\frac{1}{2}\,dt =$

$= \frac{1}{2}\int \frac{1}{1+t^2}\,dt = \frac{1}{2}\operatorname{arctg} t = \frac{1}{2}\operatorname{arctg} x^2 + C$