

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow \infty} \left[\frac{\sqrt[3]{1+2x-27x^3}}{1+3x} \right]$
2. $\lim_{x \rightarrow \infty} \left(\frac{5x^2 - x - 1}{5x^2} \right)^{2x}$
3. $\lim_{x \rightarrow 0} \frac{\cos 2x - e^{-x} - x}{x \sin x}$
4. $\lim_{x \rightarrow \infty} \frac{2^x - 3^x}{5^x - 4^x}$
5. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - x})$

Solución:

1. $\lim_{x \rightarrow \infty} \left[\frac{\sqrt[3]{1+2x-27x^3}}{1+3x} \right] = -1$
2. $\lim_{x \rightarrow \infty} \left(\frac{5x^2 - x - 1}{5x^2} \right)^{2x} = e^{-2/5}$
3. $\lim_{x \rightarrow 0} \frac{\cos 2x - e^{-x} - x}{x \sin x} = -\frac{5}{2}$
4. $\lim_{x \rightarrow 0} \frac{2^x - 3^x}{5^x - 4^x} = 0$
5. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - x}) = \frac{1}{2}$

Problema 2 Calcular las siguientes integrales:

1. $\int \left(\frac{x^3 - 3\sqrt[3]{x^2} - x}{x^2} + 2e^x \right) dx$
2. $\int x^3 e^{4x^4+8} dx$
3. $\int \frac{x^4}{1 + (4 - 2x^5)^2} dx$
4. $\int e^{2x} \cos x dx$

$$5. \int x^3 \ln x \, dx$$

Solución:

$$1. \int \left(\frac{x^3 - 3\sqrt[3]{x^2} - x}{x^2} + 2e^x \right) = \frac{x^2}{2} - 9x^{-1/3} - \ln|x| + 2e^x + C$$

$$2. \int x^3 e^{4x^4+8} \, dx = \frac{1}{16} e^{4x^4+8} + C$$

$$3. \int \frac{x^4}{1 + (4 - 2x^5)^2} = -\frac{1}{10} \arctan(4 - 2x^5) + C$$

$$4. \int e^{2x} \cos x \, dx = \frac{e^{2x}(\sin x + 2 \cos x)}{5} + C$$

$$5. \int x^3 \ln x \, dx = \frac{x^4}{16} (4 \ln x - 1) + C$$