

Problema 1 Calcular los siguientes límites:

$$1. \lim_{x \rightarrow \infty} \frac{3x^4 + x^3 - 7x - 7}{8x^4 + 9x - 5}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x}}}$$

$$3. \lim_{x \rightarrow 0} \left(\frac{\sqrt{9+x} - \sqrt{9-x}}{9x} \right)$$

$$4. \lim_{x \rightarrow \infty} \left(1 + 4x^3 \right)^{2/x^3}$$

$$5. \lim_{x \rightarrow \infty} \left(\frac{5x-7}{5x+1} \right)^{x+2}$$

$$6. \lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x^2}$$

$$7. \lim_{x \rightarrow 0} \frac{x - \sin x}{x \sin x}$$

$$8. \lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{2}{x^2 - 1} \right)$$

Solución:

$$1. \lim_{x \rightarrow \infty} \frac{3x^4 + x^3 - 7x - 7}{8x^4 + 9x - 5} = \frac{3}{8}$$

$$2. \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x}}} = 1$$

$$3. \lim_{x \rightarrow 0} \left(\frac{\sqrt{9+x} - \sqrt{9-x}}{9x} \right) = \frac{1}{27}$$

$$4. \lim_{x \rightarrow \infty} \left(1 + 4x^3 \right)^{2/x^3} = e^0 = 1$$

$$5. \lim_{x \rightarrow \infty} \left(\frac{5x-7}{5x+1} \right)^{x+2} = e^{-8/5}$$

$$6. \lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x^2} = 0$$

$$7. \lim_{x \rightarrow 0} \frac{x - \sin x}{x \sin x} = 0$$

$$8. \lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{2}{x^2 - 1} \right) = 1$$

Problema 2 Calcular las siguientes integrales:

$$1. \int \left(\frac{x^2 + 2\sqrt[3]{x} - x}{x^2} - 2e^x \right)$$

$$2. \int \left(\frac{x^2 - 3\sqrt[4]{x^3} - x}{x^2} + 3e^x \right)$$

Solución:

$$1. \int \left(\frac{x^2 + 2\sqrt[3]{x} - x}{x^2} - 2e^x \right) = x - 3x^{-2/3} - \ln x - 2e^x + C$$

$$2. \int \left(\frac{x^2 - 3\sqrt[4]{x^3} - x}{x^2} + 3e^x \right) = x + 12x^{-1/4} - \ln x + 3e^x + C$$