

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow 0} (5x + 1)^{1/x}$
2. $\lim_{x \rightarrow \infty} \left(\frac{2x^2 - x + 1}{2x^2 - 1} \right)^{3x}$
3. $\lim_{x \rightarrow \infty} (\sqrt{5x^2 - x + 2} - \sqrt{5x^2 + x - 1})$
4. $\lim_{x \rightarrow 6} \frac{\sqrt{2x^2 - 1} - \sqrt{11x + 5}}{x - 6}$
5. $\lim_{x \rightarrow 0} \frac{\cos 2x - e^{2x}}{\sin^2 x - xe^{2x}}$

Solución:

1. $\lim_{x \rightarrow 0} (5x + 1)^{1/x} = e^5$
2. $\lim_{x \rightarrow \infty} \left(\frac{2x^2 - x + 1}{2x^2 - 1} \right)^{3x} = e^{-3/2}$
3. $\lim_{x \rightarrow \infty} (\sqrt{5x^2 - x + 2} - \sqrt{5x^2 + x - 1}) = -\frac{\sqrt{5}}{5}$
4. $\lim_{x \rightarrow 6} \frac{\sqrt{2x^2 - 1} - \sqrt{11x + 5}}{x - 6} = \frac{13\sqrt{71}}{142}$
5. $\lim_{x \rightarrow 0} \frac{\cos 2x - e^{2x}}{\sin^2 x - xe^{2x}} = 2$

Problema 2 Calcular las siguientes integrales:

1. $\int (x + 5)e^x dx$
2. $\int 2xe^{7x^2-1} dx$
3. $\int e^x \cos(2x) dx$
4. $\int x^2(x^3 + 2)^{12} dx$

5. $\int \frac{3x}{5x^2+2} dx$

Solución:

1. $\int (x+5)e^x dx = e^x(x+4) + C$

2. $\int 2xe^{7x^2-1} dx = \frac{1}{7}e^{7x^2-1} + C$

3. $\int e^x \cos(2x) dx = e^x \left(\frac{\cos(2x) + 2\sin(2x)}{5} \right) + C$

4. $\int x^2(x^3+2)^{12} dx = \frac{(x^3+2)^{13}}{39} + C$

5. $\int \frac{3x}{5x^2+2} dx = \frac{3}{10} \ln(5x^2+2) + C$