

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow 0} (\cos x)^{1/\sin x}$
2. $\lim_{x \rightarrow \infty} \left(\sqrt{4x^2 - x + 1} - \sqrt{4x^2 + x - 1} \right)$
3. $\lim_{x \rightarrow \infty} \frac{3^x - 2^x}{5^x - 2^x}$
4. $\lim_{x \rightarrow 0} \frac{x^2 + 1 - \cos x}{x \sin x}$
5. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 1}{x^2 + 5} \right)^{x-1}$

Solución:

1. $\lim_{x \rightarrow 0} (\cos x)^{1/\sin x} = 1$
2. $\lim_{x \rightarrow \infty} \left(\sqrt{4x^2 - x + 1} - \sqrt{4x^2 + x - 1} \right) = -\frac{1}{2}$
3. $\lim_{x \rightarrow \infty} \frac{3^x - 2^x}{5^x - 2^x} = 0$
4. $\lim_{x \rightarrow 0} \frac{x^2 + 1 - \cos x}{x \sin x} = \frac{3}{2}$
5. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 1}{x^2 + 5} \right)^{x-1} = 1$

Problema 2 Calcular las siguientes integrales:

1. $\int x^2 e^{2x} dx$
2. $\int x \sin x dx$
3. $\int \frac{x^3 - 1}{x^2 - 4} dx$
4. $\int 3x^2 e^{5x^3+8} dx$
5. $\int \frac{3x}{1 + 16x^4} dx$

Solución:

$$1. \int x^2 e^{2x} dx = e^{2x} \frac{2x^2 - 2x + 1}{4} + C$$

$$2. \int x \sin x dx = \sin x - x \cos x + C$$

$$3. \int \frac{x^3 - 1}{x^2 - 4} dx = \frac{x^2}{2} + \frac{9}{4} \ln |x + 2| + \frac{7}{4} \ln |x - 2| + C$$

$$4. \int 3x^2 e^{5x^3+8} dx = \frac{1}{5} e^{5x^3+8} + C$$

$$5. \int \frac{3x}{1 + 16x^4} dx = \frac{3}{8} \arctan(4x^2) + C$$