

Problema 1 Calcular los siguientes límites:

- a) $\lim_{x \rightarrow 0} \frac{x - \sin x}{\tan x - \sin x}$ (Castilla La Mancha 2005)
- b) Si $f(x) = x^3 e^{-x}$ calcular $\lim_{x \rightarrow \infty} f(x)$ y $\lim_{x \rightarrow -\infty} f(x)$ (Islas Baleares 2005)
- c) $\lim_{x \rightarrow 3} \frac{\cos(\pi x)}{x}$ (La Rioja 2005)
- d) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - \sqrt{x^2 - x})$ (Madrid 2005)
- e) $\lim_{x \rightarrow \infty} x \left[\arctan(e^x) - \frac{\pi}{2} \right]$ (Madrid 2005)
- f) $\lim_{x \rightarrow 0} \frac{4x + \sin 2x}{\sin 3x}$ (Zaragoza)
- g) $\lim_{x \rightarrow \infty} \left(\frac{2x-1}{2x} \right)^{x+2}$
- h) $\lim_{x \rightarrow 0} x^x$
- i) $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 + x - 1}}{2x + 3}$
- j) $\lim_{x \rightarrow 0} \frac{\sqrt{3x+1} - \sqrt{x+1}}{x}$

Solución:

- a) $\lim_{x \rightarrow 0} \frac{x - \sin x}{\tan x - \sin x} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 0} \frac{1 - \cos x}{1 + \tan^2 x - \cos x} = \left[\frac{0}{0} \right] =$
 $\lim_{x \rightarrow 0} \frac{\sin x}{2 \tan x (1 + \tan^2 x) + \sin x} = \lim_{x \rightarrow 0} \frac{\sin x}{2 \tan x + 2 \tan^3 x + \sin x} = \left[\frac{0}{0} \right] =$
 $\lim_{x \rightarrow 0} \frac{\cos x}{2(1 + \tan^2 x) + 6 \tan^2 x (1 + \tan^2 x) + \cos x} = \frac{1}{3}$
- b) $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} x^3 e^{-x} = [\infty \cdot 0] = \lim_{x \rightarrow \infty} \frac{x^3}{e^x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{3x^2}{e^x} =$
 $\left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{6x}{e^x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{6}{e^x} = 0$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} x^3 e^{-x} = \lim_{x \rightarrow -\infty} -x^3 e^x = -\infty$$

$$\text{c) } \lim_{x \rightarrow 3} \frac{\cos(\pi x)}{x} = -\frac{1}{3}$$

d)

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - \sqrt{x^2 - x}) &= [\infty - \infty] = \\ \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + x} - \sqrt{x^2 - x})(\sqrt{x^2 + x} + \sqrt{x^2 - x})}{\sqrt{x^2 + x} + \sqrt{x^2 - x}} &= \\ \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + x})^2 - (\sqrt{x^2 - x})^2}{\sqrt{x^2 + x} + \sqrt{x^2 - x}} &= \lim_{x \rightarrow \infty} \frac{2x}{\sqrt{x^2 + x} + \sqrt{x^2 - x}} = \\ \lim_{x \rightarrow \infty} \frac{\frac{2x}{x}}{\sqrt{\frac{x^2 + x}{x^2}} + \sqrt{\frac{x^2 - x}{x^2}}} &= \frac{2}{2} = 1 \end{aligned}$$

e)

$$\begin{aligned} \lim_{x \rightarrow \infty} x \left[\arctan(e^x) - \frac{\pi}{2} \right] &= [0 \cdot \infty] = \lim_{x \rightarrow \infty} \frac{\arctan(e^x) - \frac{\pi}{2}}{1/x} = \left[\frac{0}{0} \right] = \\ \lim_{x \rightarrow \infty} \frac{\frac{e^x}{1+e^{2x}}}{-\frac{1}{x^2}} &= \lim_{x \rightarrow \infty} \frac{-x^2 e^x}{1+e^{2x}} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{-2xe^x - x^2 e^x}{2e^{2x}} = \\ \lim_{x \rightarrow \infty} \frac{-2x - x^2}{2e^x} &= \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{-2 - 2x}{2e^x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{-2}{2e^x} = 0 \\ \text{f) } \lim_{x \rightarrow 0} \frac{4x + \sin 2x}{\sin 3x} &= \left[\frac{0}{0} \right] = \lim_{x \rightarrow 0} \frac{4 + 2 \cos 2x}{3 \cos 3x} = \frac{6}{3} = 2 \\ \text{g) } \lim_{x \rightarrow \infty} \left(\frac{2x-1}{2x} \right)^{x+2} &= e^{-1/2} \\ \text{h) } \lim_{x \rightarrow 0} x^x = \lambda &\implies \lim_{x \rightarrow 0} x \ln x = \ln \lambda \\ \lim_{x \rightarrow 0} x \ln x &= \lim_{x \rightarrow 0} \frac{\ln x}{1/x} = \left[\frac{-\infty}{\pm \infty} \right] = \lim_{x \rightarrow 0} \frac{1/x}{-1/x^2} = \lim_{x \rightarrow 0} (-x) = 0 \\ \ln \lambda = 0 &\implies \lambda = 1 \\ \text{i) } \lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 + x - 1}}{2x + 3} &= \frac{\sqrt{3}}{2} \\ \text{j) } \lim_{x \rightarrow 0} \frac{\sqrt{3x+1} - \sqrt{x+1}}{x} &= 1 \end{aligned}$$