EJERCICIOS: INTEGRALES INMEDIATAS.

Resuelve las siguientes integrales inmediatas:

1. $\int (3x^3 - 5x^2 + 3x + 4)\, dx$

2. $\int (\operatorname{sen} x + 5\cos x - 1)\, dx$

3. $\int \operatorname{tg}^2 x\, dx$

4. $\int (\sqrt{x} - 3)\, dx$

5. $\int \frac{2}{\sqrt{x}}\, dx$

6. $\int \frac{x^3 - 3x^2 + 5x}{x}\, dx$

7. $\int (5x - 2)^2\, dx$

8. $\int \frac{(3x-1)^2}{3x}\, dx$

9. $\int (2\sqrt{x} - \sqrt[3]{x} - x^5)\, dx$

10. $\int \left(\frac{5}{x} - \frac{x}{5}\right) dx$

11. $\int \frac{2}{1+x^2}\, dx$

12. $\int (x-5)(x+2)\, dx$

13. $\int \frac{5e^x + e^{2x}}{e^x}\, dx$

14. $\int e^{ax+b}\, dx$

15. $\int \cos(ax+b)\, dx$

16. $\int \left(\frac{a}{x} - b\right) dx$ $\quad a, b \in \mathbb{R}$ en los tres últimos ejercicios.

Resuelve las siguientes integrales, eligiendo un cambio de variable adecuado:

1. $\int \operatorname{sen} x \cos x\, dx$

2. $\int \frac{arc\,tg\,x}{1+x^2}\,dx$

3. $\int (x^2-x-1)^5\cdot(2\mathrm{x}-1)\,dx$

4. $\int \frac{x}{x^2-1}\,dx$

5. $\int \frac{4\mathrm{x}-3}{2\mathrm{x}^2-3\mathrm{x}-14}\,dx$

6. $\int \frac{\mathrm{tg}\,x}{\cos^2 x}\,dx$

7. $\int 2\mathrm{x}\cdot\cos(x^2)\,dx$

8. $\int (1-\cos x)^3\cdot \mathrm{sen}\,x\,dx$

9. $\int \frac{1}{x\ln x}\,dx$

10. $\int \mathrm{tg}\,x\,dx$

11. $\int x^2\cdot e^{x^3}\,dx$

12. $\int \frac{2\mathrm{x}}{1+x^4}\,dx$

13. $\int \frac{e^x}{5+e^x}\,dx$

14. $\int \frac{\mathrm{sen}\,x}{e^{\cos x}}\,dx$

Resuelve, integrando por partes:

1. $\int e^x\,\mathrm{sen}\,x\,dx$

2. $\int (5\mathrm{x}-2)\,\mathrm{sen}\,x\,dx$

3. $\int \ln 3\mathrm{x}\,dx$

4. $\int x\cos x\,dx$

5. $\int x\cdot e^x\,dx$

6. $\int x^2\cdot e^x\,dx$

7. $\int (2\text{x}+4)e^{2\text{x}+4}\,dx$

8. $\int arc\,tg\,x\,dx$

9. $\int arc\,sen\,x\,dx$

10. $\int \frac{x}{e^x}\,dx$

Resuelve las siguientes integrales racionales:

1. $\int \frac{dx}{x^2-1}$

2. $\int \frac{x-1}{x^2+x-6}\,dx$

3. $\int \frac{x-5}{(x-1)^3}\,dx$

4. $\int \frac{x+2}{x^3-2\text{x}^2}\,dx$

5. $\int \frac{1}{(x-1)(x+2)^2}\,dx$

6. $\int \frac{x^3}{x-2}\,dx$

Resuelve, eligiendo el método más adecuado:

1. $\int \frac{x^2-1}{x^2+1}\,dx$

2. $\int \frac{\ln x}{x}\,dx$

3. $\int \frac{x^2}{1+x^6}\,dx$

4. $\int \frac{\ln x}{x^2}\,dx$

5. $\int (\cos 2\text{x} - \text{sen}\,3\text{x})\,dx$

6. $\int x\cdot\sqrt[4]{1-x^2}\,dx$

7. $\int \frac{\operatorname{sen} x}{1+\cos^2 x}\,dx$

8. $\int \frac{x+1}{\sqrt{x-1}}\,dx$

9. $\int \operatorname{sen} 2\mathrm{x}\,dx$

10. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

11. $\int \frac{arc\,tg^2 x}{1+x^2}\,dx$

12. $\int \frac{dx}{1+16\mathrm{x}^2}$

13. $\int \frac{x^4}{x-1}\,dx$

14. $\int \frac{\operatorname{sen}^3 x}{\cos x}\,dx$

15. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

16. $\int \frac{\ln(\ln x)}{x}\,dx$

17. $\int \frac{\cos(\ln x)}{x}\,dx$

18. $\int e^{2\mathrm{x}}\cdot\operatorname{sen} e^{2\mathrm{x}}\,dx$

19. $\int \frac{4\mathrm{x}^3-9\mathrm{x}^2+5\mathrm{x}-1}{2\mathrm{x}^4-6\mathrm{x}^3+5\mathrm{x}^2-2\mathrm{x}+7}\,dx$

20. $\int \frac{x-1}{\sqrt{1-x^2}}\,dx$

SOLUCIONES

Integrales inmediatas

1. $\int (3x^3-5x^2+3x+4)\,dx = \frac{3x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 4x + C$

2. $\int (\operatorname{sen} x + 5\cos x - 1)\,dx = -\cos x + 5\operatorname{sen} x - x + C$

3. $\int \operatorname{tg}^2 x\,dx = \int (1+\operatorname{tg}^2 x - 1)\,dx = \int (1+\operatorname{tg}^2 x)\,dx - \int 1\,dx =$

$= \operatorname{tg} x - x + C$

4. $\int (\sqrt{x} - 3)\,dx = \frac{x^{1/2}}{1/2} - 3x + C = 2\sqrt{x} - 3x + C$

5. $\int \frac{2}{\sqrt{x}}\,dx = 2 \cdot \frac{x^{1/2}}{1/2} + C = 4\sqrt{x} + C$

6. $\int \frac{x^3-3x^2+5x}{x}\,dx = \int (x^2-3x+5)\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + 5x + C$

7. $\int (5x-2)^2\,dx = \frac{1}{5}\int 5(5x-2)^2\,dx = \frac{1}{5} \cdot \frac{(5x-2)^3}{3} + C = \frac{(5x-2)^3}{15} + C$

$\int (5x-2)^2\,dx = \int (25x^2-20x+4)\,dx = \frac{25x^3}{3} - 10x^2 + 4x + C$

8. $\int \frac{(3x-1)^2}{3x}\,dx = \int \frac{9x^2-6x+1}{3x}\,dx = \int \left(3x - 2 + \frac{1}{3x}\right)dx =$

$= \frac{3x^2}{2} - 2x + \frac{1}{3}\ln|x| + C$

9. $\int (2\sqrt{x} - \sqrt[3]{x} - x^5)\,dx = \int (2x^{1/2} - x^{1/3} - x^5)\,dx =$

$= 2 \cdot \frac{x^{3/2}}{3/2} - \frac{x^{4/3}}{4/3} - \frac{x^6}{6} + C = \frac{4}{3}\sqrt{x^3} - \frac{3}{4}\sqrt[3]{x^4} - \frac{x^6}{6} + C$

10. $\int \left(\frac{5}{x} - \frac{x}{5} \right) dx = 5 \ln|x| - \frac{1}{5} \cdot \frac{x^2}{2} + C = 5 \ln|x| - \frac{x^2}{10} + C$

11. $\int \frac{2}{1+x^2} dx = 2 \operatorname{arctg} x + C$

12. $\int (x-5)(x+2)\, dx = \int (x^2 - 3x - 10)\, dx = \frac{x^3}{3} - \frac{3x^2}{2} - 10x + C$

13. $\int \frac{5e^x + e^{2x}}{e^x} dx = \int \frac{5e^x}{e^x} dx + \int \frac{e^{2x}}{e^x} dx = \int 5\, dx + \int e^x dx =$

$= 5x + e^x + C$

14. $\int e^{ax+b} dx = \frac{1}{a} \int a \cdot e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$

15. $\int \cos(ax+b)\, dx = \frac{1}{a} \int a \cdot \cos(ax+b)\, dx = \frac{1}{a} \operatorname{sen}(ax+b) + C$

16. $\int \left(\frac{a}{x} - b \right) dx = a \cdot \ln|x| - bx + C$

<u>Integrales por cambio de variable</u>

1. $\int \operatorname{sen} x \cos x \, dx = \int t\, dt = \frac{t^2}{2} + C = \frac{\operatorname{sen}^2 x}{2} + C$

$t = \operatorname{sen} x \Rightarrow dt = \cos x\, dx$

2. $\int \frac{\operatorname{arctg} x}{1+x^2} dx = \int t\, dt = \frac{t^2}{2} + C = \frac{\operatorname{arctg}^2 x}{2} + C$

$t = \operatorname{arctg} x \Rightarrow dt = \frac{1}{1+x^2} dx$

3. $\int (x^2 - x - 1)^5 \cdot (2x-1)\, dx = \int t^5 dt = \frac{t^6}{6} + C = \frac{(x^2-x-1)^6}{6} + C$

$t = x^2 - x - 1 \Rightarrow dt = 2x - 1$

4. $\int \frac{x}{x^2-1}\,dx = \frac{1}{2}\int \frac{2x}{x^2-1}\,dx = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t|+C =$

$t=x^2-1 \Rightarrow dt=2x\,dx$

$= \frac{1}{2}\ln|x^2-1|+C$

5. $\int \frac{4x-3}{2x^2-3x-14}\,dx = \int \frac{1}{t}\,dt = \ln|t|+C = \ln|2x^2-3x-14|+C$

$t=2x^2-3x-14 \Rightarrow dt=4x-3$

6. $\int \frac{tg\,x}{\cos^2 x}\,dx = \int t\,dt = \frac{t^2}{2}+C = \frac{tg^2 x}{2}+C$

$t=tg\,x \Rightarrow dt=\frac{1}{\cos^2 x}\,dx$

7. $\int 2x\cos x^2\,dx = \int \cos t\,dt = sen\,t+C = sen\,x^2+C$

$t=x^2 \Rightarrow dt=2x\,dx$

8. $\int (1-\cos x)^3 \cdot sen\,x\,dx = \int t^3\,dt = \frac{t^4}{4}+C = \frac{(1-\cos x)^4}{4}+C$

$t=1-\cos x \Rightarrow dt=sen\,x\,dx$

9. $\int \frac{1}{x\ln x}\,dx = \int \frac{1}{t}\,dt = \ln|t|+C = \ln|\ln x|+C$

$t=\ln x \Rightarrow dt=\frac{1}{x}\,dx$

10. $\int tg\,x\,dx = \int \frac{sen\,x}{\cos x}\,dx = -\int \frac{1}{t}\,dt = -\ln|t|+C =$

$t=\cos x \Rightarrow dt=-sen\,x\,dx$

$= -\ln|\cos x|+C$

11. $\int x^2 e^{x^3}\,dx = \frac{1}{3}\int e^t\,dt = \frac{1}{3}e^t+C = \frac{1}{3}e^{x^3}+C$

$t=x^3 \Rightarrow dt=3x^2\,dx$

12. $\int \frac{2x}{1+x^4}\,dx = \int \frac{2x}{1+(x^2)^2}\,dx = \int \frac{1}{1+t^2}\,dt = \operatorname{arctg} t + C =$

$t = x^2 \Rightarrow dt = 2x\,dx$

$= \operatorname{arctg}(x^2) + C$

13. $\int \frac{e^x}{5+e^x}\,dx = \int \frac{1}{t}\,dt = \ln|t| + C = \ln(5+e^x) + C$

$t = 5+e^x \Rightarrow dt = e^x dx$

14. $\int \frac{\operatorname{sen} x}{e^{\cos x}}\,dx = -\int \frac{1}{e^t}\,dt = -\int e^{-t}\,dt = e^{-t} + C =$

$t = \cos x \Rightarrow dt = -\operatorname{sen} x\,dx$

$= e^{-\cos x} + C = \frac{1}{e^{\cos x}} + C$

Integrales por partes:

1. $\int e^x \operatorname{sen} x\,dx = -e^x \cos x + \int e^x \cos x\,dx =$

$u = e^x \Rightarrow du = e^x dx$

$dv = \operatorname{sen} x\,dx \Rightarrow v = -\cos x$

$u = e^x \Rightarrow du = e^x dx$

$dv = \cos x\,dx \Rightarrow v = \operatorname{sen} x$

$= -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x\,dx$

Llamando $I = \int e^x \operatorname{sen} x\,dx$, hemos obtenido:

$I = -e^x \cos x + e^x \operatorname{sen} x - I$

$2I = -e^x \cos x + e^x \operatorname{sen} x$

$I = \frac{-e^x \cos x + e^x \operatorname{sen} x}{2}$

Por tanto, $\int e^x \operatorname{sen} x\,dx = \frac{e^x(\operatorname{sen} x - \cos x)}{2} + C$

2. $\int (5x-2)\, \text{sen}\, x\, dx = (5x-2)\cos x - \int 5\cos x\, dx =$

$u = 5x-2 \Rightarrow du = 5\,dx$

$dv = \text{sen}\, x\, dx \Rightarrow v = \cos x$

$= (5x-2)\cos x - 5\,\text{sen}\, x + C$

3. $\int \ln x\, dx = x\ln x - \int \frac{1}{\cancel{x}} \cdot \cancel{x}\, dx = x\ln x - x + C$

$u = \ln x \Rightarrow du = \frac{1}{x}\, dx$

$dv = dx \Rightarrow v = x$

4. $\int x\cos x\, dx = x\,\text{sen}\, x - \int \text{sen}\, x\, dx = x\,\text{sen}\, x + \cos x + C$

$u = x \Rightarrow du = dx$

$dv = \cos x\, dx \Rightarrow v = \text{sen}\, x$

5. $\int x e^x\, dx = x e^x - \int e^x\, dx = x e^x - e^x + C$

$u = x \Rightarrow du = dx$

$dv = e^x \Rightarrow v = e^x$

6. $\int x^2 e^x\, dx = x^2 e^x - \int 2x e^x\, dx = x^2 e^x - 2x e^x + \int 2 e^x\, dx =$

$u = x^2 \Rightarrow du = 2x\, dx$ $\quad$ $u = 2x \Rightarrow du = 2dx$

$dv = e^x dx \Rightarrow v = e^x$ $\quad$ $dv = e^x dx \Rightarrow v = e^x$

$= x^2 e^x - 2x e^x + 2e^x + C$

7. $\int (2x+4)\, e^{2x+4}\, dx = \frac{2x+4}{2}\, e^{2x+4} - \int \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot e^{2x+4}\, dx =$

$u = 2x+4 \Rightarrow du = 2dx$

$dv = e^{2x+4} dx \Rightarrow v = \frac{1}{2} e^{2x+4}$

$= (x+2)\, e^{2x+4} - \frac{1}{2} e^{2x+4} + C = \left(x + \frac{3}{2}\right) e^{2x+4} + C$

8. $\int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \int \frac{x}{1+x^2} dx = x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C$

$u = \operatorname{arctg} x \Rightarrow du = \frac{1}{1+x^2} dx$

$dv = dx \Rightarrow v = x$

9. $\int \operatorname{arc\,sen} x \, dx = x \operatorname{arc\,sen} x - \int \frac{x}{\sqrt{1-x^2}} dx = x \operatorname{arc\,sen} x - \frac{1}{2} \cdot \frac{(1-x^2)^{1/2}}{1/2} + C =$

$u = \operatorname{arc\,sen} x \Rightarrow du = \frac{1}{\sqrt{1-x^2}} dx$

$dv = dx \Rightarrow v = x$

$= x \operatorname{arc\,sen} x - \sqrt{1-x^2} + C$

10. $\int \frac{x}{e^x} dx = \int x e^{-x} dx = -x e^{-x} + \int e^{-x} dx =$

$u = x \Rightarrow du = dx$

$dv = e^{-x} dx \Rightarrow v = -e^{-x}$

$= -x e^{-x} - e^{-x} + C = \frac{-x-1}{e^x} + C$

Integrales Racionales:

1. $\int \frac{1}{x^2-1} dx$

$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1) + B(x+1)}{x^2-1}$

$1 = A(x-1) + B(x+1) \Rightarrow \begin{cases} A = -1/2 \\ B = 1/2 \end{cases}$

$\int \frac{1}{x^2-1} dx = \int \frac{-1/2}{x+1} dx + \int \frac{1/2}{x-1} dx = -\frac{1}{2} \ln|x+1| + \frac{1}{2} \ln|x-1| + C =$

$= \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + C = \ln \sqrt{\left| \frac{x-1}{x+1} \right|} + C$

2. $\int \frac{x-1}{x^2+x-6}\,dx = \int \frac{x-1}{(x-2)(x+3)}\,dx$

$$\frac{x-1}{x^2+x-6} = \frac{x-1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{A(x+3)+B(x-2)}{(x-2)(x+3)}$$

$$x-1 = A(x+3)+B(x-2) \Rightarrow \begin{cases} A = 1/5 \\ B = 2/3 \end{cases}$$

$$\int \frac{x-1}{x^2+x-6}\,dx = \int \frac{1/5}{x-2}\,dx + \int \frac{2/3}{x+3}\,dx = \frac{1}{5}\ln|x-2| + \frac{2}{3}\ln|x+3| + C$$

3. $\int \frac{x-5}{(x-1)^3}\,dx$

$$\frac{x-5}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} = \frac{A(x-1)^2+B(x-1)+C}{(x-1)^3}$$

$$x-5 = A(x-1)^2+B(x-1)+C \Rightarrow \begin{cases} A=-4 \\ B=1 \\ C=-4 \end{cases}$$

$$\int \frac{x-5}{(x-1)^3}\,dx = \int \frac{-4}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx + \int \frac{-4}{(x-1)^3}\,dx =$$

$$= -4\ln|x-1| + \frac{(x-1)^{-1}}{-1} - \frac{4(x-1)^{-2}}{-2} + C = -4\ln|x-1| - \frac{1}{x-1} + \frac{2}{(x-1)^2} + C$$

4. $\int \frac{x+2}{x^3-2x^2}\,dx = \int \frac{x+2}{x^2(x-2)}\,dx$

$$\frac{x+2}{x^3-2x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} = \frac{Ax(x-2)+B(x-2)+Cx^2}{x^2(x-2)}$$

$$x+2 = Ax(x-2)+B(x-2)+Cx^2 \Rightarrow \begin{cases} A=-1 \\ B=-1 \\ C=1 \end{cases}$$

$$\int \frac{x+2}{x^3-2x^2}\,dx = \int -\frac{1}{x}\,dx + \int \frac{-1}{x^2}\,dx + \int \frac{1}{x-2}\,dx =$$

$$= -\ln|x| - \frac{x^{-1}}{-1} + \ln|x-2| = \ln\left|\frac{x-2}{x}\right| + \frac{1}{x} + C$$

5. $\int \frac{1}{(x-1)(x+2)^2}\,dx$

$$\frac{1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{(x-1)(x+2)^2}$$

$$1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \Rightarrow \begin{cases} A = 1/9 \\ B = -1/9 \\ C = -1/3 \end{cases}$$

$$\int \frac{1}{(x-1)(x+2)^2}\,dx = \int \frac{1/9}{x-1}\,dx - \int \frac{1/9}{x+2}\,dx - \int \frac{1/3}{(x+2)^2}\,dx =$$

$$= \frac{1}{9}\ln|x-1| - \frac{1}{9}\ln|x+2| - \frac{1}{3}\cdot\frac{(x+2)^{-1}}{-1} + C =$$

$$= \frac{1}{9}\ln\left|\frac{x-1}{x+2}\right| + \frac{1}{3(x+2)} + C$$

6. $\int \frac{x^3}{x-2}\,dx = \int \left(x^2+2x+4+\frac{8}{x-2}\right)dx = \frac{x^3}{3} + x^2 + 4x + 8\ln|x-2| + C$

$$\begin{array}{l|l} x^3 & x-2 \\ \hline -x^3+2x^2 & x^2+2x+4 \\ \hline \quad 2x^2 & \\ \quad -2x^2+4x & \\ \hline \qquad 4x & \\ \qquad -4x+8 & \\ \hline \qquad\quad 8 & \end{array}$$

Integrales Varias:

1. $\int \frac{x^2-1}{x^2+1}\,dx = \int \left(1 - \frac{2}{x^2+1}\right) dx = x - 2\,\mathrm{arctg}\,x + C$

$$\begin{array}{l|l} x^2-1 & x^2+1 \\ \cline{2-2} -x^2-1 & 1 \\ \hline \quad -2 & \end{array}$$

2. $\int \frac{\ln x}{x}\,dx = \int t\,dt = \frac{t^2}{2} + C = \frac{\ln^2 x}{2} + C$

$t = \ln x \Rightarrow dt = \frac{1}{x}\,dx$

3. $\int \frac{x^2}{1+x^6}\,dx = \frac{1}{3}\int \frac{3x^2}{1+(x^3)^2}\,dx = \frac{1}{3}\int \frac{1}{1+t^2}\,dt = \frac{1}{3}\,\mathrm{arctg}\,t + C =$

$t = x^3 \Rightarrow dt = 3x^2\,dx$

$= \frac{1}{3}\,\mathrm{arctg}\,x^3 + C$

4. $\int \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} + \int \frac{1}{x}\cdot\frac{1}{x}\,dx = -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx =$

$u = \ln x \Rightarrow du = \frac{1}{x}\,dx$

$dv = \frac{1}{x^2}\,dx \Rightarrow v = -\frac{1}{x}$

$= -\frac{\ln x}{x} - \frac{1}{x} + C = -\frac{\ln x - 1}{x} + C$

5. $\int (\cos 2x - \mathrm{sen}\,3x)\,dx = \frac{1}{2}\int 2\cos 2x - \frac{1}{3}\int 3\,\mathrm{sen}\,3x\,dx =$

$= \frac{1}{2}\,\mathrm{sen}\,2x + \frac{1}{3}\cos 3x + C$

6. $\int x\cdot\sqrt[4]{1-x^2}\,dx = -\frac{1}{2}\int \sqrt[4]{t}\,dt = -\frac{1}{2}\cdot\frac{t^{5/4}}{5/4} + C = -\frac{2}{5}\sqrt[4]{t^5} + C =$

$t = 1-x^2 \Rightarrow dt = -2x\,dx$

$= -\frac{2}{5}\sqrt[4]{(1-x^2)^5} + C$

7. $\displaystyle\int \frac{\operatorname{sen} x}{1+\cos x}\,dx = -\int \frac{1}{t}\,dt = -\ln|t| + C = -\ln|1+\cos x| + C$

$t = 1+\cos x \Rightarrow dt = \operatorname{sen} x\,dx$

8. $\displaystyle\int \frac{x+1}{\sqrt{x-1}}\,dx = \int \frac{t^2+2}{\sqrt{t^2}}\cdot 2\cancel{t}\,dt = \int (2t^2+4)\,dt =$

$t^2 = x+1 \Rightarrow 2t\,dt = dx$

$\displaystyle = \frac{2t^3}{3} + 4t + C = \frac{2(\sqrt{x-1})^3}{3} + 4\sqrt{x-1} + C$

9. $\displaystyle\int \operatorname{sen} 2x\,dx = \frac{1}{2}\int 2\operatorname{sen} 2x\,dx = \frac{-1}{2}\cos 2x + C$

10. $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}} = 2\int \frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx = 2\int \cos t\,dt = -2\operatorname{sen} t + C =$

$t = \sqrt{x} \Rightarrow dt = \frac{1}{2\sqrt{x}}\,dx$

$= -2\operatorname{sen}\sqrt{x} + C$

11. $\displaystyle\int \frac{\operatorname{arctg}^2 x}{1+x^2}\,dx = \int t^2\,dt = \frac{t^3}{3} + C = \frac{\operatorname{arctg}^3 x}{3} + C$

$t = \operatorname{arctg} x \Rightarrow dt = \frac{1}{1+x^2}\,dx$

12. $\displaystyle\int \frac{dx}{1+16x^2} = \int \frac{dx}{1+(4x)^2} = \frac{1}{4}\int \frac{1}{1+t^2}\,dt = \frac{1}{4}\operatorname{arctg} t + C =$

$t = 4x \Rightarrow dt = 4\,dx$

$\displaystyle = \frac{1}{4}\operatorname{arctg}(4x) + C$

13. $\displaystyle\int \frac{x^4}{x-1}\,dx = \int \left(x^3+x^2+x+1+\frac{1}{x-1}\right)dx = \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + C$

$$
\begin{array}{l|l}
x^4 & x-1 \\ \cline{2-2}
\underline{-x^4+x^3} & x^3+x^2+x+1 \\
\quad x^3 & \\
\quad \underline{-x^3+x^2} & \\
\qquad x^2 & \\
\qquad \underline{-x^2+x} & \\
\qquad\quad x & \\
\qquad\quad \underline{x+1} & \\
\qquad\qquad 1 &
\end{array}
$$

14. $\int \frac{sen^3 x}{\cos x}\,dx = \int \frac{sen x \cdot sen^2 x}{\cos x}\,dx = \int \frac{sen x \cdot (1-\cos^2 x)}{\cos x}\,dx =$

$= \int \frac{sen x - sen x \cos^2 x}{\cos x}\,dx = \int \frac{sen x}{\cos x}\,dx - \int sen x \cos x\,dx =$

$t = \cos x \Rightarrow dt = -sen\,dx$

$= -\int \frac{1}{t}\,dt - \int t\,dt = -\ln|t| + \frac{t^2}{2} + C =$

$= -\ln|\cos x| + \frac{\cos^2 x}{2} + C$

15. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^t dt = 2e^t + C = 2e^{\sqrt{x}} + C$

$t = \sqrt{x} \Rightarrow dt = \frac{1}{2\sqrt{x}}\,dx$

16. $\int \frac{\ln(\ln x)}{x}\,dx = \int \ln t\,dt = t \cdot \ln t - \int \frac{1}{t} t\,dt =$

$t = \ln x \Rightarrow dt = \frac{1}{x}\,dx$

$u = \ln t \Rightarrow du = \frac{1}{t}\,dt$

$dv = dt \Rightarrow v = t$

$= t \cdot \ln t - t + C = \ln(\ln x) - \ln x + C$

17. $\int \frac{\cos(\ln x)}{x}\,dx = \int \cos t\,dt = sen\,t + C = sen(\ln x) + C$

$t = \ln x \Rightarrow dt = \frac{1}{x}dx$

18. $\int e^{2x} \cdot sen\,e^{2x}\,dx = \frac{1}{2}\int sen\,t\,dt = -\frac{1}{2}\cos t + C =$

$t = e^{2x} \Rightarrow dt = 2e^{2x}dx$

$= -\frac{1}{2}\cos e^{2x} + C$

19. $\int \frac{4x^3-9x^2+5x-1}{2x^4-6x^3+5x^2-2x+7}\,dx = \frac{1}{2}\int \frac{8x^3-18x^2+10x-2}{2x^4-6x^3+5x^2-2x+7}\,dx =$

$= \frac{1}{2}\ln|2x^4-6x^3+5x^2-2x+7| + C$

20. $\int \frac{x-1}{\sqrt{1-x^2}}\,dx = \int \frac{x}{\sqrt{1-x^2}}\,dx - \int \frac{1}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} - \arcsin x + C$

$* \int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt = \frac{-1}{2}\cdot\frac{t^{1/2}}{1/2} = -\sqrt{t}+C = -\sqrt{1-x^2}+C$

$t = 1-x^2 \Rightarrow dt = -2x\,dx$