1	Calcular la derivada de las siguientes funciones:

a) $y = \dfrac{x+1}{3x-5}$ b) $y = \dfrac{x^2+5}{2x-3}$ c) $y = \dfrac{10}{1+x^2}$

d) $y = \dfrac{x^3+1}{x^2-x-2}$ e) $y = (x^2 + a^2)^5$ f) $y = \sqrt{x^2+a^2}$

g) $y = \left(1+\sqrt{x+a}\right)^2$ h) $y = -\dfrac{1}{\sqrt{x-a}}$ i) $y = \left(x-\sqrt{1-x^2}\right)^2$

j) $y = \sqrt[6]{2x^5-3x^2}$ k) $y = x^2\sqrt{x^3}$ l) $y = \sqrt{\dfrac{1+x}{1-x}}$

m) $y = \dfrac{1-x}{\sqrt{1-x^2}}$ n) $y = \dfrac{a+\sqrt{x}}{a-\sqrt{x}}$ o) $y = \sqrt{\dfrac{a+x}{a-x}}$

SOLUCIÓN:

a) $y = \dfrac{x+1}{3x-5}$

$$y' = \frac{3x-5-(x+1)\cdot 3}{(3x-5)^2} = \frac{3x-5-3x-3}{(3x-5)^2} = \frac{-8}{(3x-5)^2}$$

b) $y = \dfrac{x^2+5}{2x-3}$

$$y' = \frac{2x(2x-3)-(x^2+5)\cdot 2}{(2x-3)^2} = \frac{4x^2-6x-2x^2-10}{(2x-3)^2} = \frac{2x^2-6x-10}{(2x-3)^2}$$

c) $y = \dfrac{10}{1+x^2}$

$$y' = \frac{-10\cdot 2x}{(1+x^2)^2} = \frac{-20x}{(1+x^2)^2}$$

d) $y = \dfrac{x^3+1}{x^2-x-2}$

$$y' = \frac{3x^2(x^2-x-2)-(x^3+1)(2x-1)}{(x^2-x-2)^2} = \frac{3x^4-3x^3-6x^2-2x^4+x^3-2x+1}{(x^2-x-2)^2} == \frac{x^4-2x^3-6x^2-2x+1}{(x^2-x-2)^2} =$$

$$= \frac{(x^2-4x+1)(x+1)^2}{(x-2)^2(x+1)^2} = \frac{x^2-4x+1}{(x-2)^2}$$

e) $y = (x^2 + a^2)^5$

$$y' = 5\cdot(x^2+a^2)^4\cdot 2x = 10x\cdot(x^2+a^2)^4$$

f) $y = \sqrt{x^2 + a^2}$

$$y' = \frac{2x}{2\sqrt{x^2 + a^2}} = \frac{x}{\sqrt{x^2 + a^2}}$$

g) $y = \left(1 + \sqrt{x + a}\right)^2$

$$y' = 2\left(1 + \sqrt{x + a}\right) \cdot \frac{1}{2\sqrt{x + a}} = \frac{1 + \sqrt{x + a}}{\sqrt{x + a}}$$

h) $y = -\frac{1}{\sqrt{x - a}}$

$$y' = -\frac{1}{\sqrt{x - a}} = -\frac{-\frac{1}{2\sqrt{x - a}}}{x - a} = \frac{1}{2(x - a)\sqrt{x - a}} = \frac{\sqrt{x - a}}{2(x - a)^2}$$

i) $y = \left(x - \sqrt{1 - x^2}\right)^2$

$$y' = 2\left(x - \sqrt{1 - x^2}\right)\left(1 - \frac{-2x}{2\sqrt{1 - x^2}}\right) = 2\left(x - \sqrt{1 - x^2}\right)\left(1 + \frac{x}{\sqrt{1 - x^2}}\right) = 2\left(x - \sqrt{1 - x^2}\right)\left(\frac{\sqrt{1 - x^2} + x}{\sqrt{1 - x^2}}\right) =$$

$$= 2\left(\frac{x^2 - 1 + x^2}{\sqrt{1 - x^2}}\right) = \frac{2(2x^2 - 1)}{\sqrt{1 - x^2}}$$

j) $y = \sqrt[6]{2x^5 - 3x^2}$

$$y' = \frac{1}{6\sqrt[6]{(2x^5 - 3x^2)^5}} \cdot (10x^4 - 6x) = \frac{5x^4 - 3x}{3\sqrt[6]{(2x^5 - 3x^2)^5}}$$

k) $y = x^2\sqrt{x^3}$

$$y = x^2\sqrt{x^3} = x^3\sqrt{x} = x^{\frac{7}{2}} \qquad \to y' = \frac{7}{2}x^{\frac{5}{2}} = \frac{7}{2}x^2\sqrt{x}$$

l) $y = \sqrt{\frac{1 + x}{1 - x}}$

$$y' = \frac{1}{2\sqrt{\frac{1 + x}{1 - x}}} \cdot \frac{1 - x - (1 + x)(-1)}{(1 - x)^2} = \frac{\sqrt{1 - x}}{2\sqrt{1 + x}} \cdot \frac{2}{(1 - x)^2} = \frac{1}{\sqrt{1 + x}\sqrt{1 - x}(1 - x)} = \frac{1}{\sqrt{1 - x^2}(1 - x)}$$

m) $y = \dfrac{1-x}{\sqrt{1-x^2}}$

$$y' = \dfrac{-\sqrt{1-x^2} - (1-x)\cdot\dfrac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}}{1-x^2} = \dfrac{-\sqrt{1-x^2} + (1-x)\cdot\dfrac{x}{\sqrt{1-x^2}}}{1-x^2} = \dfrac{\dfrac{-(1-x^2)+x-x^2}{\sqrt{1-x^2}}}{1-x^2} =$$

$$= \dfrac{x-1}{(1-x^2)\sqrt{1-x^2}} = \dfrac{(x-1)\sqrt{1-x^2}}{(1-x^2)^2} = \dfrac{\sqrt{1-x^2}}{(x+1)^2(x-1)}$$

n) $y = \dfrac{a+\sqrt{x}}{a-\sqrt{x}}$

$$y' = \dfrac{a+\sqrt{x}}{a-\sqrt{x}} = \dfrac{\dfrac{1}{2\sqrt{x}}\left(a-\sqrt{x}\right)-\left(a+\sqrt{x}\right)\left(-\dfrac{1}{2\sqrt{x}}\right)}{\left(a-\sqrt{x}\right)^2} = \dfrac{a-\sqrt{x}+a+\sqrt{x}}{2\sqrt{x}\left(a-\sqrt{x}\right)^2} = \dfrac{a}{\sqrt{x}\left(a-\sqrt{x}\right)^2}$$

o) $y = \sqrt{\dfrac{a+x}{a-x}}$

$$y' = \sqrt{\dfrac{a+x}{a-x}}\;\dfrac{1}{2\sqrt{\dfrac{a+x}{a-x}}}\cdot\dfrac{(a-x)-(a+x)(-1)}{(a-x)^2} = \dfrac{\sqrt{a-x}}{2\sqrt{a+x}}\cdot\dfrac{2a}{(a-x)^2} = \dfrac{a}{(a-x)\sqrt{a^2-x^2}}$$

2 Calcular la derivada de las siguientes funciones trigonométricas:

a) $y = \text{sen}^3 x$	b) $y = \text{sen}\ x^3$	c) $y = \text{sen}\dfrac{x}{2}\cos\dfrac{x}{2}$
d) $y = \dfrac{\cos x}{1-\text{sen}\,x}$	e) $y = \dfrac{\text{sen}\,x}{1+\text{tg}^2 x}$	f) $y = \sqrt{1+\text{sen}\,x}$
g) $y = \dfrac{\cos x+\text{sen}\,x}{\cos x-\text{sen}\,x}$	h) $y = \dfrac{1-\text{tg}^2 x}{1+\text{tg}^2 x}$	i) $y = \text{arcsen}\dfrac{x}{2}$
j) $y = \text{ar}\cos\sqrt{1-x^2}$	k) $y = \text{arctg}\dfrac{2x}{1-x^2}$	l) $y = \text{arctg}\dfrac{a+x}{1-ax}$

SOLUCIÓN:

a) $y = \text{sen}^3 x$

$y' = 3\ \text{sen}^2 x \cos x$

b) $y = \text{sen}\ x^3$

$y' = \cos x^3 \cdot 3x^2 = 3x^2 \cdot \cos x^3$

c) $y = \operatorname{sen}\frac{x}{2}\cos\frac{x}{2}$

$$y = \operatorname{sen}\frac{x}{2}\cos\frac{x}{2} = \frac{1}{2}\operatorname{sen}x \rightarrow y' = \frac{1}{2}\cos x$$

d) $y = \frac{\cos x}{1-\operatorname{sen}x}$

$$y' = \frac{-\operatorname{sen}x(1-\operatorname{sen}x)-\cos x(-\cos x)}{(1-\operatorname{sen}x)^2} = \frac{-\operatorname{sen}x+\operatorname{sen}^2x+\cos^2 x}{(1-\operatorname{sen}x)^2} = \frac{1-\operatorname{sen}x}{(1-\operatorname{sen}x)^2} = \frac{1}{1-\operatorname{sen}x}$$

e) $y = \frac{\operatorname{sen}x}{1+\operatorname{tg}^2x}$

$$y' = \frac{\cos x\left(\cancel{1+\operatorname{tg}^2x}\right)-\operatorname{sen}x\cdot 2\operatorname{tg}x\left(\cancel{1+\operatorname{tg}^2x}\right)}{\left(1+\operatorname{tg}^2x\right)^{\cancel{2}}} = \frac{\cos x-2\operatorname{sen}x\operatorname{tg}x}{1+\operatorname{tg}^2x} = \frac{\cos x-2\frac{\operatorname{sen}^2 x}{\cos x}}{\frac{1}{\cos^2 x}} =$$

$$= \frac{\frac{\cos^2 x-2\operatorname{sen}^2 x}{\cos x}}{\frac{1}{\cos^2 x}} = \cos x\left(1-\operatorname{sen}^2 x-2\operatorname{sen}^2 x\right) = \cos x\left(1-3\operatorname{sen}^2 x\right)$$

f) $y = \sqrt{1+\operatorname{sen}x}$

$$y' = \frac{1}{2\sqrt{1+\operatorname{sen}x}}\cdot\cos x = \frac{\cos x}{2\sqrt{1+\operatorname{sen}x}}$$

g) $y = \frac{\cos x+\operatorname{sen}x}{\cos x-\operatorname{sen}x}$

$$y' = \frac{(-\operatorname{sen}x+\cos x)(\cos x-\operatorname{sen}x)-(\cos x+\operatorname{sen}x)(-\operatorname{sen}x-\cos x)}{(\cos x-\operatorname{sen}x)^2} =$$

$$= \frac{(\cos x-\operatorname{sen}x)^2+(\cos x+\operatorname{sen}x)^2}{(\cos x-\operatorname{sen}x)^2} = \frac{(1-2\operatorname{sen}x\cos x)+(1+2\operatorname{sen}x\cos x)}{1-2\operatorname{sen}x\cos x} = \frac{2}{1-\operatorname{sen}2x}$$

h) $y = \frac{1-\operatorname{tg}^2x}{1+\operatorname{tg}^2x}$

$$y' = \frac{-2\operatorname{tg}x\left(1+\operatorname{tg}^2x\right)^2-\left(1-\operatorname{tg}^2x\right)2\operatorname{tg}x\left(1+\operatorname{tg}^2x\right)}{\left(1+\operatorname{tg}^2x\right)^2} = \frac{-2\operatorname{tg}x\left(1+\operatorname{tg}^2x+1-\operatorname{tg}^2x\right)}{1+\operatorname{tg}^2x} = \frac{-4\operatorname{tg}x}{1+\operatorname{tg}^2x} = -4\operatorname{sen}x\cos x$$

i) $y = \operatorname{arcsen}\frac{x}{2}$

$$y' = \frac{1}{2}\cdot\frac{1}{\sqrt{1-\frac{x^2}{4}}} = \frac{1}{2}\cdot\frac{1}{\frac{\sqrt{4-x^2}}{2}} = \frac{1}{\sqrt{4-x^2}}$$

j) $y = \arccos\sqrt{1-x^2}$

$$y' = \frac{-1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\cdot\frac{-2x}{2\sqrt{1-x^2}} = \frac{x}{x\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

k) $y = \text{arctg}\dfrac{2x}{1-x^2}$

$$y' = \frac{1}{1+\left(\dfrac{2x}{1-x^2}\right)^2}\cdot\frac{2(1-x^2)-2x(-2x)}{(1-x^2)^2} = \frac{1}{\dfrac{1+x^4-2x^2+4x^2}{(1-x^2)^2}}\cdot\frac{2-2x^2+4x^2}{(1-x^2)^2} = \frac{2x^2+2}{x^4+2x^2+1} =$$

$$= \frac{2(x^2+1)}{(x^2+1)^2} = \frac{2}{x^2+1}$$

l) $y = \text{arctg}\dfrac{a+x}{1-ax}$

$$y' = \frac{1}{1+\left(\dfrac{a+x}{1-ax}\right)^2}\cdot\frac{(1-ax)-(a+x)(-a)}{(1-ax)^2} = \frac{1}{\dfrac{1+a^2x^2-2ax+a^2+x^2+2ax}{(1-ax)^2}}\cdot\frac{1-ax+a^2+ax}{(1-ax)^2} =$$

$$= \frac{1+a^2}{1+a^2+a^2x^2+x^2} = \frac{1+a^2}{(1+a^2)+(a^2+1)x^2} = \frac{1+a^2}{(1+a^2)(x^2+1)} = \frac{1}{x^2+1}$$

3 Calcular la derivada de las siguientes funciones logarítmicas y exponenciales:

a) $y = \dfrac{1+\ln x}{x}$ b) $y = (x-1)\cdot e^x$ c) $y = \ln(\ln x)$

d) $y = \log_a(3x^2+5)$ e) $y = \ln\left(\dfrac{1+x^2}{1-x^2}\right)$ f) $y = \ln\left(\dfrac{e^x}{1+e^x}\right)$

g) $y = \ln\left(x+\sqrt{1+x^2}\right)$ h) $y = (x^2-2x)\cdot e^x$ i) $y = \dfrac{e^x-1}{e^x+1}$

j) $y = 2a\ln(a+x)-x$ k) $y = \dfrac{x^3}{e^x}$ l) $y = e^x\cdot\cos x$

m) $y = \dfrac{\ln x}{x^3}$ n) $y = \ln\left(\dfrac{x}{\sqrt{1+x^2}}\right)$ o) $y = 2^x\cdot x^2$

SOLUCIÓN:

a) $y = \dfrac{1+\ln x}{x}$

$$y' = \frac{1+\ln x}{x} = \frac{1}{x^2}\cdot\left(\frac{1}{x}\cdot x-(1+\ln x)\right) = \frac{-\ln x}{x^2}$$

b) $y = (x - 1) \cdot e^x$

$$y' = e^x + (x - 1)\, e^x = x\, e^x$$

c) $y = \ln(\ln x)$

$$y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \cdot \ln x}$$

d) $y = \log_a(3x^2 + 5)$

$$y' = \quad \frac{6x}{3x^2 + 5} \cdot \frac{1}{\ln a}$$

e) $y = \ln\left(\frac{1+x^2}{1-x^2}\right)$

$$y' = \frac{1-x^2}{1+x^2} \cdot \frac{2x(1-x^2)-(1+x^2)(-2x)}{(1-x^2)^2} = \frac{1}{1+x^2} \cdot \frac{2x(1-x^2+1+x^2)}{1-x^2} = \frac{4x}{1-x^4}$$

f) $y = \ln\left(\frac{e^x}{1+e^x}\right)$

$$y' = \frac{1+e^x}{e^x} \cdot \frac{e^x(1+e^x)-e^x \cdot e^x}{(1+e^x)^2} = \frac{1}{e^x} \cdot \frac{e^x(1+e^x-e^x)}{1+e^x} = \frac{1}{1+e^x}$$

g) $y = \ln\left(x + \sqrt{1+x^2}\right)$

$$y' = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}\right) = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$$

h) $y = (x^2 - 2x) \cdot e^x$

$$y' = (2x - 2)\, e^x + (x^2 - 2x) \cdot e^x = (\,2x - 2 + x^2 - 2x) \cdot e^x = (x^2 - 2) \cdot e^x$$

i) $y = \frac{e^x - 1}{e^x + 1}$

$$y' = \frac{e^x - 1}{e^x + 1} = \frac{e^x(e^x+1)-(e^x-1)e^x}{(e^x+1)^2} = \frac{e^x(e^x+1-e^x+1)}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}$$

j) $y = 2a \ln (a + x) - x$

$$y' = \frac{2a}{a+x} - 1 = \frac{2a-a-x}{a+x} = \frac{a-x}{a+x}$$

k) $y = \frac{x^3}{e^x}$

$$y' = \frac{3x^2 \cdot e^x - x^3 \cdot e^x}{e^{2x}} = \frac{(3x^2 - x^3) \cdot e^x}{e^{2x}} = \frac{3x^2 - x^3}{e^x}$$

l) $y = e^x \cdot \cos x$

$y' = e^x \cdot \cos x + e^x \cdot (-\text{sen } x) = e^x (\cos x - \text{sen } x)$

m) $y = \dfrac{\ln x}{x^3}$

$$y' = \frac{\frac{1}{x}\cdot x^3 - \ln x(3x^2)}{x^6} = \frac{x^2(1-3\ln x)}{x^6} = \frac{1-3\ln x}{x^4}$$

n) $y = \ln\left(\dfrac{x}{\sqrt{1+x^2}}\right)$

$$y' = \frac{\sqrt{1+x^2}}{x}\cdot\frac{\sqrt{1+x^2} - \frac{x}{\cancel{2}\sqrt{1+x^2}}\cdot\cancel{2}x}{1+x^2} = \frac{\sqrt{1+x^2}}{x}\cdot\frac{1+x^2-x^2}{(1+x^2)\sqrt{1+x^2}} = \frac{1}{x(1+x^2)}$$

o) $y = 2^x \cdot x^2$

$y' = 2^x \cdot \ln 2 \cdot x^2 + 2^x \cdot 2x = 2^x\, x\,(x \ln 2 + 2)$

4 Calcular la derivada de las siguientes funciones, aplicando la derivación logarítmica:

a) $y = x^x$ b) $y = x^{\ln x}$ c) $y = x^{\cos x}$ d) $y = (\text{tg } x)^{\text{tg } x}$

SOLUCIÓN:

a) $y = x^x$

$$\ln y = \ln x^x \rightarrow \ln y = x \ln x \rightarrow \frac{y'}{y} = \ln x + x\cdot\frac{1}{x} \rightarrow y' = y(1+\ln x) \quad \rightarrow y' = (1+\ln x)\cdot x^x$$

b) $y = x^{\ln x}$

$$\ln y = \ln x^{\ln x} \rightarrow \ln y = (\ln x)^2 \rightarrow \frac{y'}{y} = 2\ln x\cdot\frac{1}{x} \rightarrow y' = y\left(\frac{2}{x}\cdot\ln x\right) \quad \rightarrow y' = \left(\frac{2}{x}\cdot\ln x\right)\cdot x^{\ln x}$$

c) $y = x^{\cos x}$

$$\ln y = \cos x \cdot \ln x \rightarrow \frac{y'}{y} = -\text{sen}\, x\cdot\ln x + \cos x\cdot\frac{1}{x}$$

$$y' = y\left(-\text{sen}\, x\cdot\ln x + \cos x\cdot\frac{1}{x}\right) \quad \rightarrow y' = \left(-\text{sen}\, x\cdot\ln x + \cos x\cdot\frac{1}{x}\right)\cdot x^{\cos x}$$

d) $y = (\text{tg } x)^{\text{tg } x}$

$$\ln y = \text{tg}x \cdot \ln(\text{tg } x) \rightarrow \frac{y'}{y} = (1+\text{tg}^2 x)\cdot\ln(\text{tg}\, x) + \text{tg}\, x\cdot\frac{1}{\text{tg}\, x}(1+\text{tg}^2 x) \rightarrow \frac{y'}{y} = (1+\text{tg}^2 x)\cdot(1+\ln(\text{tg}\, x))$$

$$y' = (1+\text{tg}^2 x)\cdot(1+\ln(\text{tg}\, x))\cdot\text{tg}\, x^{\text{tg}\, x}$$

4	Calcular la derivada de las siguientes funciones, simplificando el resultado:

a) $y = \left(x-\sqrt{1-x^2}\right)^2$ b) $y = \sqrt{\dfrac{1-\cos x}{1+\cos x}}$ c) $y = \ln\sqrt{\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}$

d) $y = \ln\left(\dfrac{x^2}{\sqrt{1-x^2}}\right)$ e) $y = \ln\left(\dfrac{1+\operatorname{sen} x}{\operatorname{tg} x}\right)$ f) $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

SOLUCIÓN:

a) $y = \left(x-\sqrt{1-x^2}\right)^2$

$$y' = 2\left(x-\sqrt{1-x^2}\right)\left(1-\frac{-2x}{2\sqrt{1-x^2}}\right) = 2\left(x-\sqrt{1-x^2}\right)\left(\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}}\right) = \frac{2\left(x^2-1+x^2\right)}{\sqrt{1-x^2}} = \frac{2\left(2x^2-1\right)}{\sqrt{1-x^2}}$$

b) $y = \sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$$y' = \frac{1}{2\sqrt{\dfrac{1-\cos x}{1+\cos x}}}\cdot\frac{\operatorname{sen} x(1+\cos x)-(1-\cos x)(-\operatorname{sen} x)}{(1+\cos x)^2} = \frac{\sqrt{1+\cos x}}{2\sqrt{1-\cos x}}\cdot\frac{2\operatorname{sen} x}{(1+\cos x)^2} =$$

Multiplicamos y dividimos por $\sqrt{1+\cos x}$

$$= \frac{1+\cos x}{\sqrt{1-\cos^2 x}}\cdot\frac{\operatorname{sen} x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

c) $y = \ln\sqrt{\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}$

$$y' = \frac{1}{\sqrt{\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}}\cdot\frac{1}{2\sqrt{\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}}\cdot\frac{\cos x(1-\operatorname{sen} x)-(1+\operatorname{sen} x)(-\cos x)}{(1-\operatorname{sen} x)^2} = \frac{1}{2\left(\dfrac{1+\operatorname{sen} x}{1-\operatorname{sen} x}\right)}\cdot\frac{2\cos x}{(1-\operatorname{sen} x)^2} =$$

$$= \frac{1}{(1+\operatorname{sen} x)}\cdot\frac{\cos x}{(1-\operatorname{sen} x)} = \frac{\cos x}{1-\operatorname{sen}^2 x} = \frac{1}{\cos x}$$

d) $y = \ln\left(\dfrac{x^2}{\sqrt{1-x^2}}\right)$

$$y' = \frac{\sqrt{1-x^2}}{x^2}\cdot\frac{2x\sqrt{1-x^2} - x^2\cdot\dfrac{-2x}{2\sqrt{1-x^2}}}{\left(\sqrt{1-x^2}\right)^2} = \frac{\sqrt{1-x^2}}{x^2}\cdot\frac{2x\left(1-x^2\right)+x^3}{\left(1-x^2\right)\sqrt{1-x^2}} = \frac{1}{x^2}\cdot\frac{2x-x^3}{\left(1-x^2\right)} = \frac{2-x^2}{\left(1-x^2\right)x}$$

e) $y = \ln\left(\frac{1+\operatorname{sen} x}{\operatorname{tg} x}\right)$

$$y' = \frac{\operatorname{tg} x}{1+\operatorname{sen} x}\cdot\frac{\cos x\cdot\operatorname{tg} x-(1+\operatorname{sen} x)(1+\operatorname{tg}^2 x)}{\operatorname{tg}^2 x} = \frac{1}{1+\operatorname{sen} x}\cdot\frac{\operatorname{sen} x-\frac{1+\operatorname{sen} x}{\cos^2 x}}{\operatorname{tg} x} = \frac{1}{1+\operatorname{sen} x}\cdot\frac{\operatorname{sen} x-\frac{1+\operatorname{sen} x}{1-\operatorname{sen}^2 x}}{\operatorname{tg} x} =$$

$$= \frac{1}{1+\operatorname{sen} x}\cdot\frac{\operatorname{sen} x-\frac{1}{1-\operatorname{sen} x}}{\operatorname{tg} x} = \frac{1}{1+\operatorname{sen} x}\cdot\frac{\operatorname{sen} x-\operatorname{sen}^2 x-1}{\operatorname{tg} x(1-\operatorname{sen} x)} = \frac{\operatorname{sen} x-\operatorname{sen}^2 x-1}{\operatorname{tg} x(1-\operatorname{sen}^2 x)} = \frac{\operatorname{sen} x-\operatorname{sen}^2 x-1}{\cos x\cdot\operatorname{sen} x}$$

f) $y = \frac{e^x-e^{-x}}{e^x+e^{-x}}$

$$y' = \frac{e^x-e^{-x}}{e^x+e^{-x}}\frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2} = \frac{(e^{2x}+e^{-2x}+2)-(e^{2x}+e^{-2x}-2)}{(e^x+e^{-x})^2} = \frac{4}{(e^x+e^{-x})^2}$$

5 Calcular la derivada de las siguientes funciones, simplificando el resultado:

a) $y = \arcsin\left(\frac{x^2-1}{x^2}\right)$ b) $y = \operatorname{arctg}\left(\frac{1+\operatorname{sen} x}{\cos x}\right)$ c) $y = \arccos\left(\frac{1-x^2}{1+x^2}\right)$

d) $y = \arcsin\left(\frac{x}{\sqrt{1+x^2}}\right)$ e) $y = \operatorname{arctg}\left(x+\sqrt{1+x^2}\right)$ f) $y = \operatorname{arctg}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

g) $y = \arcsin\left(\frac{2\operatorname{tg} x}{1+\operatorname{tg}^2 x}\right)$ h) $y = \operatorname{arctg}\left(\frac{\cos x}{\sqrt{1+\operatorname{sen}^2 x}}\right)$ i) $y = \arcsin\left(\frac{1+x^2}{x}\right)$

j) $y = \arcsin\sqrt{1-x^2}$ k) $y = \operatorname{arctg}\sqrt{\frac{1-x}{1+x}}$ l) $y = \operatorname{ar}\cos\sqrt{1-\frac{1}{x^2}}$

m) $y = \operatorname{arctg}(3^x)$ n) $y = \operatorname{arctg}(2x+1)$ o) $y = \operatorname{arctg}\left(\frac{x+a}{1-ax}\right)$

SOLUCIÓN:

a) $y = \arcsin\left(\frac{x^2-1}{x^2}\right)$

$$y' = \frac{1}{\sqrt{1-\left(\frac{x^2-1}{x^2}\right)^2}}\cdot\frac{2x\cdot x^2-(x^2-1)\cdot 2x}{x^4} = \frac{x^4}{\sqrt{x^4-(x^2-1)^2}}\cdot\frac{2x}{x^4} = \frac{2x}{\sqrt{2x^2-1}}$$

b) $y = \operatorname{arctg}\left(\frac{1+\operatorname{sen} x}{\cos x}\right)$

$$y' = \frac{1}{1+\left(\frac{1+\operatorname{sen} x}{\cos x}\right)^2}\cdot\frac{\cos^2 x-(1+\operatorname{sen} x)(-\operatorname{sen} x)}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x+(1+\operatorname{sen} x)^2}\cdot\frac{\cos^2 x+\operatorname{sen} x+\operatorname{sen}^2 x}{\cos^2 x} =$$

$$= \frac{1}{\cos^2 x + 1 + \operatorname{sen}^2 x + 2\operatorname{sen}x} \cdot (1 + \operatorname{sen}x) = \frac{1}{2 + 2\operatorname{sen}x} \cdot (1 + \operatorname{sen}x) = \frac{1}{2}$$

c) $y = \arccos\left(\frac{1 - x^2}{1 + x^2}\right)$

$$y' = -\frac{1}{\sqrt{1 - \left(\frac{1 - x^2}{1 + x^2}\right)^2}} \cdot \frac{-2x \cdot (1 + x^2) - (1 - x^2) \cdot 2x}{(1 + x^2)^2} = -\frac{1 + x^2}{\sqrt{(1 + x^2)^2 - \left(1 - x^2\right)^2}} \cdot \frac{-4x}{(1 + x^2)^2} =$$

$$= \frac{1 + x^2}{\sqrt{4x^2}} \cdot \frac{4x}{(1 + x^2)^2} = \frac{1}{2x} \cdot \frac{4x}{1 + x^2} = \frac{2}{1 + x^2}$$

d) $y = \operatorname{arcsen}\left(\frac{x}{\sqrt{1 + x^2}}\right)$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{x}{\sqrt{1 + x^2}}\right)^2}} \cdot \frac{\sqrt{1 + x^2} - x \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{1 + x^2}}}{\left(\sqrt{1 + x^2}\right)^2} = \frac{\sqrt{1 + x^2}}{\sqrt{1 + x^2 - x^2}} \cdot \frac{1 + x^2 - x^2}{\left(\sqrt{1 + x^2}\right)^2 \sqrt{1 + x^2}} = \frac{1}{1 + x^2}$$

e) $y = \operatorname{arctg}\left(x + \sqrt{1 - x^2}\right)$

$$y' = \frac{1}{1 + \left(x + \sqrt{1 + x^2}\right)^2} \cdot \left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{1 + x^2}}\right) = \frac{1}{1 + \left(x + \sqrt{1 + x^2}\right)^2} \cdot \frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2}} =$$

$$= \frac{1}{1 + x^2 + 1 + x^2 + 2x\sqrt{1 + x^2}} \cdot \frac{x + \sqrt{1 + x^2}}{\sqrt{1 + x^2}} = \frac{1}{2\left(1 + x^2 + x\sqrt{1 + x^2}\right)} \cdot \frac{x + \sqrt{1 + x^2}}{\sqrt{1 + x^2}} =$$

$$= \frac{1}{2\sqrt{1 + x^2}\left(\sqrt{1 + x^2} + x\right)} \cdot \frac{x + \sqrt{1 + x^2}}{\sqrt{1 + x^2}} = \frac{1}{2\left(1 + x^2\right)}$$

f) $y = \operatorname{arctg}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$

$$y' = \frac{1}{1 + \frac{x^2}{a^2 - x^2}} \cdot \frac{\sqrt{a^2 - x^2} - x \cdot \frac{-2x}{2\sqrt{a^2 - x^2}}}{a^2 - x^2} = \frac{a^2 - x^2}{a^2 - x^2 + x^2} \cdot \frac{\sqrt{a^2 - x^2} - x \cdot \frac{-2x}{2\sqrt{a^2 - x^2}}}{a^2 - x^2} =$$

$$= \frac{1}{a^2} \cdot \frac{a^2 - x^2 + x^2}{\sqrt{a^2 - x^2}} = \frac{1}{\sqrt{a^2 - x^2}}$$

g) $y = \operatorname{arcsen}\left(\frac{2\operatorname{tg}x}{1 + \operatorname{tg}^2 x}\right)$

$$y' = \frac{1}{\sqrt{1-\left(\frac{2\,tg\,x}{1+tg^2x}\right)^2}} \cdot \frac{2(1+tg^2x)^{\not 2} - 2\,tg\,x \cdot 2\,tg\,x \cdot (\cancel{1+tg^2x})}{(1+tg^2x)^{\not 2}} = \frac{1+tg^2x}{\sqrt{(1+tg^2x)^2 - 4\,tg^2\,x}} \cdot \frac{2+2tg^2x-4tg^2\,x}{1+tg^2x} =$$

$$= \frac{2-2\,tg^2\,x}{\sqrt{1+tg^4x-2tg^2x}} = \frac{2(1-tg^2x)}{\sqrt{(1-tg^2x)^2}} = \frac{2(1-tg^2x)}{(1-tg^2x)} = 2$$

h) $y = \operatorname{arctg}\left(\frac{\cos x}{\sqrt{1+sen^2x}}\right)$

$$y' = \frac{1}{1+\frac{\cos^2 x}{1+sen^2x}} \cdot \frac{-sen\,x\sqrt{1+sen^2x} - \cos x \cdot \frac{2sen\,x\cos x}{2\sqrt{1+sen^2x}}}{1+sen^2x} =$$

$$= \frac{1+sen^2x}{1+sen^2x+\cos^2 x} \cdot \frac{-sen\,x(1+sen^2x) - sen\,x\cos^2 x}{(1+sen^2x)\sqrt{1+sen^2x}} == \frac{1}{2} \cdot \frac{-sen\,x(1+sen^2x+\cos^2 x)}{\sqrt{1+sen^2x}} =$$

$$= \frac{1}{2} \cdot \frac{-2sen\,x}{\sqrt{1+sen^2x}} = \frac{-sen\,x}{\sqrt{1+sen^2x}}$$

i) $y = \operatorname{arcsen}\left(\frac{1+x^2}{x}\right)$

$$y' = \frac{1}{\sqrt{1-\left(\frac{1+x^2}{x}\right)^2}} \cdot \frac{2x^2-(1+x^2)}{x^2} = \frac{x}{\sqrt{x^2-1-2x^2-x^4}} \cdot \frac{x^2-1}{x^2} = \frac{x^2-1}{x\sqrt{-1-x^2-x^4}}$$

j) $y = \operatorname{arcsen}\sqrt{1-x^2}$

$$y' = \frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}} \cdot \frac{-2x}{2\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-1+x^2}\sqrt{1-x^2}} = \frac{-1}{\sqrt{1-x^2}}$$

k) $y = \operatorname{arctg}\sqrt{\frac{1-x}{1+x}}$

$$y' = \frac{1}{1+\frac{1-x}{1+x}} \cdot \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \cdot \frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{\cancel{1+x}}{1+x+1-x} \cdot \frac{\sqrt{1+x}}{2\sqrt{1-x}} \cdot \frac{-1-x-1+x}{(1+x)^{\not 2}} = \frac{-1}{2\sqrt{1-x}\sqrt{1+x}} = \frac{-1}{2\sqrt{1-x^2}}$$

l) $y = \operatorname{ar\cos}\sqrt{1-\frac{1}{x^2}}$

$$y' = \frac{-1}{\sqrt{1-\left(\sqrt{\frac{x^2-1}{x^2}}\right)^2}} \cdot \frac{1}{2\sqrt{\frac{x^2-1}{x^2}}} \cdot \frac{2x^3-2x(x^2-1)}{x^4} = \frac{-1}{\sqrt{\frac{x^2-x^2+1}{x^2}}} \cdot \frac{x}{2\sqrt{x^2-1}} \cdot \frac{2x}{x^4} = \frac{-1}{x\sqrt{x^2-1}}$$

m) $y = \operatorname{arctg}(3^x)$

$$y' = \frac{1}{1+3^{2x}} \cdot 3^x \cdot \ln 3 = \frac{3^x \cdot \ln 3}{1+3^{2x}}$$

n) $y = \operatorname{arctg}(2x+1)$

$$y' = \frac{2}{1+(2x+1)^2} = \frac{2}{1+4x^2+4x+1} = \frac{1}{2x^2+2x+1}$$

o) $y = \operatorname{arctg}\left(\frac{x+a}{1-ax}\right)$

$$y' = \frac{1}{1+\left(\frac{x+a}{1-ax}\right)^2} \cdot \frac{1-ax-(x+a)(-a)}{(1-ax)^2} = \frac{(1-ax)^2}{1+a^2x^2+a^2+x^2} \cdot \frac{1+a^2}{(1-ax)^2} = \frac{1+a^2}{(1+a^2)(1+x^2)} = \frac{1}{1+x^2}$$

6 Calcular la derivada de las siguientes funciones, simplificando el resultado:

a) $y = \frac{x}{2(1-x^2)} + \frac{1}{4} \cdot \ln\left(\frac{1-x}{1+x}\right)$

b) $y = -\frac{\cos x}{2\operatorname{sen}^2 x} + \frac{1}{2}\ln\left(\operatorname{tg}\frac{x}{2}\right)$

c) $y = \frac{1}{2}\operatorname{tg}^2 x + \ln(\cos x)$

d) $y = \ln\left(x+\sqrt{1+x^2}\right) - \frac{\sqrt{1+x^2}}{x}$

e) $y = \frac{3}{2}\operatorname{arcsen}(x-1) - \frac{x+3}{2}\sqrt{2x-x^2}$

f) $y = \operatorname{arctg}\sqrt{\frac{1+x}{1-x}} + \frac{1}{2}\operatorname{arcsen}\left(\sqrt{1-x^2}\right)$

g) $y = \sqrt{2x-x^2} - \arccos(1-x)$

h) $y = \sqrt{x^2-1} + x^2 \operatorname{arc\,sen}\left(\frac{1}{x}\right)$

i) $y = x\sqrt{a^2-x^2} + a^2 \cdot \operatorname{arc\,sen}\left(\frac{x}{a}\right)$

j) $y = 2\sqrt{ax-a^2} + a \cdot \arccos\left(\frac{x-2a}{x}\right)$

k) $y = \sqrt{x^2+x} - \ln\left(\sqrt{x}+\sqrt{x+1}\right)$

l) $y = \ln\left(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x}\right)$

m) $y = \ln\left(\frac{\sqrt{e^x+1}+1}{\sqrt{e^x+1}-1}\right)$

n) $y = \ln\left(\sqrt[4]{\frac{\operatorname{sen} x}{1-\cos x}}\right)$

o) $y = \frac{3-2x}{2} + \ln\left(e^x \cdot \operatorname{tg} x\right)$

p) $y = \sqrt{a^2-x^2} - a \cdot \arccos\left(\frac{x}{a}\right)$

q) $y = \operatorname{arctg}\left(\frac{1-e^x}{1+e^x}\right)$

r) $y = \ln\left(\sqrt[3]{\frac{1+\cos x}{\operatorname{sen} x}}\right)$

s) $y = \ln\left(x^2 \cdot \sqrt{1-x^2}\right)$

t) $y = \ln\left(\frac{x^2}{\sqrt{1-x^2}}\right)$

u) $y = \left(\frac{1+\operatorname{sen} x}{\cos x}\right)^2$

v) y = -ctg x + ln (sen x)

SOLUCIÓN:

a) $y = \dfrac{x}{2(1-x^2)} + \dfrac{1}{4}\cdot\ln\left(\dfrac{1-x}{1+x}\right)$

$$y' = \frac{1}{2}\cdot\frac{1-x^2-x(-2x)}{(1-x^2)^2} + \frac{1}{4}\cdot\left(\frac{1+x}{1-x}\right)\cdot\frac{-(1+x)-(1-x)}{(1+x)^2} = \frac{1}{2}\cdot\frac{1+x^2}{(1-x^2)^2} + \frac{1}{4}\cdot\left(\frac{1}{1-x}\right)\cdot\frac{-2}{1+x} =$$

$$= \frac{1}{2}\cdot\frac{1+x^2}{(1-x^2)^2} - \frac{1}{2}\cdot\frac{1}{1-x^2} = \frac{1}{2}\cdot\frac{1+x^2-1+x^2}{(1-x^2)^2} = \frac{x^2}{(1-x^2)^2}$$

b) $y = -\dfrac{\cos x}{2\text{sen}^2 x} + \dfrac{1}{2}\ln\left(\text{tg}\dfrac{x}{2}\right)$

$$y' = -\frac{1}{2}\cdot\frac{-\text{sen}x\cdot\text{sen}^2x - \cos x\cdot 2\text{sen}x\cos x}{\text{sen}^4 x} + \frac{1}{2}\cdot\frac{1}{\text{tg}\frac{x}{2}}\cdot\left(1+\text{tg}^2\frac{x}{2}\right)\cdot\frac{1}{2} = \frac{\text{sen}^2x + 2\cos^2 x}{2\text{sen}^3 x} + \frac{1}{4}\cdot\frac{1}{\text{tg}\frac{x}{2}}\cdot\frac{1}{\cos^2\frac{x}{2}} =$$

$$= \frac{1+\cos^2 x}{2\text{sen}^3 x} + \frac{1}{4\cdot\text{sen}\frac{x}{2}\cos\frac{x}{2}} = \frac{1+\cos^2 x}{2\text{sen}^3 x} + \frac{1}{2\text{sen}x} = \frac{1+\cos^2 x + \text{sen}^2 x}{2\text{sen}^3 x} = \frac{1}{\text{sen}^3 x}$$

c) $y = \dfrac{1}{2}\text{tg}^2 x + \ln(\cos x)$

$$y' = \frac{1}{2}\cdot 2\cdot\text{tg}x\left(1+\text{tg}^2 x\right) + \frac{-\text{sen}x}{\cos x} = \text{tg}x(1+\text{tg}^2x-1) = \text{tg}^3 x$$

d) $y = \ln\left(x+\sqrt{1+x^2}\right) - \dfrac{\sqrt{1+x^2}}{x}$

$$y' = \frac{1}{x+\sqrt{1+x^2}}\cdot\left(1+\frac{2x}{2\sqrt{1+x^2}}\right) - \frac{\frac{2x^2}{2\sqrt{1+x^2}} - \sqrt{1+x^2}}{x^2} = \frac{1}{x+\sqrt{1+x^2}}\cdot\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right) - \frac{x^2-1-x^2}{x^2\sqrt{1+x^2}} =$$

$$= \frac{1}{\sqrt{1+x^2}} + \frac{1}{x^2\sqrt{1+x^2}} = \frac{x^2+1}{x^2\sqrt{1+x^2}} = \frac{\sqrt{1+x^2}}{x^2}$$

e) $y = \dfrac{3}{2}\text{arcsen}(x-1) - \dfrac{x+3}{2}\sqrt{2x-x^2}$

$$y' = \frac{3}{2}\cdot\frac{1}{\sqrt{1-(x-1)^2}} - \frac{1}{2}\left(\sqrt{2x-x^2} + (x+3)\cdot\frac{2-2x}{2\sqrt{2x-x^2}}\right) = \frac{3}{2\sqrt{2x-x^2}} - \frac{1}{2}\left(\frac{2x-x^2+(x+3)\cdot(1-x)}{\sqrt{2x-x^2}}\right) =$$

$$= \frac{3}{2\sqrt{2x-x^2}} - \frac{1}{2}\cdot\frac{3-2x^2}{\sqrt{2x-x^2}} = \frac{x^2}{\sqrt{2x-x^2}}$$

f) $y = \text{arctg}\sqrt{\dfrac{1+x}{1-x}} + \dfrac{1}{2}\text{arcsen}\left(\sqrt{1-x^2}\right)$

$$y' = \frac{1}{1+\frac{1+x}{1-x}}\cdot\frac{1}{2\sqrt{\frac{1+x}{1-x}}}\cdot\frac{1-x-(1+x)(-1)}{(1-x)^2} + \frac{1}{2}\cdot\frac{1}{\sqrt{1-(1-x^2)}}\cdot\frac{-2x}{2\sqrt{1-x^2}} =$$

$$= \frac{1-x}{2}\cdot\frac{\sqrt{1-x}}{2\sqrt{1+x}}\cdot\frac{2}{(1-x)^2} - \frac{1}{2}\cdot\frac{1}{x}\cdot\frac{x}{\sqrt{1-x^2}} = \frac{1}{2\sqrt{1-x^2}} - \frac{1}{2\sqrt{1-x^2}} = 0$$

g) $y = \sqrt{2x - x^2} - \arccos(1-x)$

$$y' = \frac{2-2x}{2\sqrt{2x-x^2}} - \frac{1}{\sqrt{1-(1-x)^2}} = \frac{1-x}{\sqrt{2x-x^2}} - \frac{1}{\sqrt{2x-x^2}} = \frac{-x}{\sqrt{2x-x^2}}$$

h) $y = \sqrt{x^2-1} + x^2 \operatorname{arc\,sen}\left(\frac{1}{x}\right)$

$$y' = \frac{2x}{2\sqrt{x^2-1}} + 2x\cdot \operatorname{arc\,sen}\left(\frac{1}{x}\right) + \frac{\cancel{x^2}}{\sqrt{1-\frac{1}{x^2}}}\cdot\left(-\frac{1}{\cancel{x^2}}\right) = \frac{x}{\sqrt{x^2-1}} + 2x\cdot \operatorname{arc\,sen}\left(\frac{1}{x}\right) - \frac{x}{\sqrt{x^2-1}} = 2x\cdot \operatorname{arc\,sen}\left(\frac{1}{x}\right)$$

i) $y = x\sqrt{a^2-x^2} + a^2\cdot \operatorname{arc\,sen}\left(\frac{x}{a}\right)$

$$y' = \sqrt{a^2-x^2} + x\cdot\frac{-\cancel{2}x}{\cancel{2}\sqrt{a^2-x^2}} + a^{\cancel{2}}\cdot\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\cdot\frac{1}{\cancel{a}} = \sqrt{a^2-x^2} - \frac{x^2}{\sqrt{a^2-x^2}} + \frac{a^2}{\sqrt{a^2-x^2}} =$$

$$= \sqrt{a^2-x^2} + \frac{a^2-x^2}{\sqrt{a^2-x^2}} = 2\sqrt{a^2-x^2}$$

j) $y = 2\sqrt{ax-a^2} + a\cdot\arccos\left(\frac{x-2a}{x}\right)$

$$y' = 2\frac{a}{2\sqrt{ax-a^2}} + a\cdot\frac{-1}{\sqrt{1-\left(\frac{x-2a}{x}\right)^2}}\cdot\frac{x-(x-2a)}{x^2} = \frac{a}{\sqrt{ax-a^2}} - \frac{ax}{\sqrt{x^2-x^2+4ax-4a^2}}\cdot\frac{2a}{x^2} =$$

$$= \frac{a}{\sqrt{ax-a^2}} - \frac{2a^2}{2x\sqrt{ax-a^2}} = \frac{ax-a^2}{\sqrt{ax-a^2}} = \sqrt{ax-a^2}$$

k) $y = \sqrt{x^2+x} - \ln\left(\sqrt{x} + \sqrt{x+1}\right)$

$$y' = \frac{2x+1}{2\sqrt{x^2+x}} - \frac{1}{\sqrt{x}+\sqrt{x+1}}\cdot\left(\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x+1}}\right) = \frac{2x+1}{2\sqrt{x^2+x}} - \frac{1}{\sqrt{x}+\sqrt{x+1}}\cdot\frac{2\left(\sqrt{x}+\sqrt{x+1}\right)}{4\sqrt{x^2+x}} =$$

$$= \frac{2x+1}{2\sqrt{x^2+x}} - \frac{1}{2\sqrt{x^2+x}} = \frac{2x}{2\sqrt{x^2+x}} = \frac{x}{\sqrt{x^2+x}}$$

l) $y=\ln\left(\dfrac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x}\right)$

$$y'=\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}-x}\cdot\frac{\left(\dfrac{2x}{2\sqrt{x^2+1}}-1\right)\cdot\left(\sqrt{x^2+1}+x\right)-\left(\sqrt{x^2+1}-x\right)\cdot\left(\dfrac{2x}{2\sqrt{x^2+1}}+1\right)}{\left(\sqrt{x^2+1}+x\right)^2}=$$

$$=\frac{1}{\sqrt{x^2+1}-x}\cdot\frac{\left(\dfrac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right)\cdot\left(\sqrt{x^2+1}+x\right)-\left(\sqrt{x^2+1}-x\right)\cdot\left(\dfrac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}+x}=$$

$$=\frac{\left(\dfrac{x^2-x^2-1}{\sqrt{x^2+1}}\right)-\left(\dfrac{x^2+1-x^2}{\sqrt{x^2+1}}\right)}{x^2+1-x^2}=\frac{-2}{\sqrt{x^2+1}}$$

m) $y=\ln\left(\dfrac{\sqrt{e^x+1}+1}{\sqrt{e^x+1}-1}\right)$

$$y'=\frac{\cancel{\sqrt{e^x+1}-1}}{\sqrt{e^x+1}+1}\cdot\frac{\dfrac{e^x}{2\sqrt{e^x+1}}\cdot\left(\sqrt{e^x+1}-1\right)-\left(\sqrt{e^x+1}+1\right)\cdot\dfrac{e^x}{2\sqrt{e^x+1}}}{\left(\sqrt{e^x+1}-1\right)^{\cancel{2}}}=$$

$$=\frac{1}{\sqrt{e^x+1}+1}\cdot\frac{\dfrac{e^x}{2\sqrt{e^x+1}}\cdot\left(\sqrt{e^x+1}-1-\sqrt{e^x+1}-1\right)}{\sqrt{e^x+1}-1}=\frac{\dfrac{-\cancel{2}e^x}{\cancel{2}\sqrt{e^x+1}}}{e^x+1-1}=\frac{-e^x}{e^x\sqrt{e^x+1}}=\frac{-1}{\sqrt{e^x+1}}$$

n) $y=\ln\left(\sqrt[4]{\dfrac{\operatorname{sen}x}{1-\cos x}}\right)$

$$y'=\frac{1}{\sqrt[4]{\dfrac{\operatorname{sen}x}{1-\cos x}}}\cdot\frac{1}{4\sqrt[4]{\left(\dfrac{\operatorname{sen}x}{1-\cos x}\right)^3}}\cdot\frac{\cos x(1-\cos x)-\operatorname{sen}x\cdot\operatorname{sen}x}{(1-\cos x)^2}=\frac{1}{\dfrac{\operatorname{sen}x}{1-\cos x}}\cdot\frac{\cos x-1}{(1-\cos x)^2}=\frac{-1}{\operatorname{sen}x}$$

o) $y=\dfrac{3-2x}{2}+\ln\left(e^x\cdot\operatorname{tg}x\right)$

$$y'=-1+\frac{e^x\cdot\operatorname{tg}x+e^x\cdot(1+\operatorname{tg}^2x)}{e^x\cdot\operatorname{tg}x}=-1+1+\frac{1+\operatorname{tg}^2x}{\operatorname{tg}x}=\frac{1}{\cos^2x}:\frac{\operatorname{sen}x}{\cos x}=\frac{1}{\operatorname{sen}x\cdot\cos x}$$

p) $y=\sqrt{a^2-x^2}-a\cdot\arccos\left(\dfrac{x}{a}\right)$

$$y'=\frac{-2x}{2\sqrt{a^2-x^2}}-\frac{-a}{\sqrt{1-\left(\dfrac{x}{a}\right)^2}}\cdot\left(\frac{1}{a}\right)=\frac{-x}{\sqrt{a^2-x^2}}+\frac{a}{\sqrt{a^2-x^2}}=\frac{(a-x)\sqrt{a^2-x^2}}{a^2-x^2}=\frac{\sqrt{a^2-x^2}}{a+x}$$

q) $y = \text{arctg}\left(\frac{1-e^x}{1+e^x}\right)$

$$y' = \frac{1}{1+\left(\frac{1-e^x}{1+e^x}\right)^2}\cdot\frac{-e^x\cdot(1+e^x)-(1-e^x)\cdot e^x}{(1+e^x)^2} = \frac{-e^x\cdot(1+e^x+1-e^x)}{(1+e^x)^2+(1-e^x)^2} = \frac{-2e^x}{2+2e^{2x}} = \frac{-e^x}{1+e^{2x}}$$

r) $y = \ln\left(\sqrt[3]{\frac{1+\cos x}{\text{sen}\,x}}\right)$

$$y' = \frac{1}{\sqrt[3]{\frac{1+\cos x}{\text{sen}\,x}}}\cdot\frac{1}{3\sqrt[3]{\left(\frac{1+\cos x}{\text{sen}\,x}\right)^2}}\cdot\frac{-\text{sen}^2x-(1+\cos x)\cos x}{\text{sen}^2x} = \frac{\text{sen}x}{3(1+\cos x)}\cdot\frac{-1-\cos x}{\text{sen}^2x} = \frac{-1}{3\text{sen}x}$$

s) $y = \ln\left(x^2\cdot\sqrt{1-x^2}\right)$

$$y' = \frac{1}{x^2\sqrt{1-x^2}}\cdot\left(2x\sqrt{1-x^2}+x^2\cdot\frac{-2x}{2\sqrt{1-x^2}}\right) = \frac{1}{x^2\sqrt{1-x^2}}\cdot\left(2x\sqrt{1-x^2}-\frac{x^3}{\sqrt{1-x^2}}\right) = \frac{1}{x^2\sqrt{1-x^2}}\cdot\left(\frac{2x-3x^3}{\sqrt{1-x^2}}\right) =$$

$$= \frac{2-3x^2}{x(1-x^2)}$$

t) $y = \ln\left(\frac{x^2}{\sqrt{1-x^2}}\right)$

$$y' = \frac{\sqrt{1-x^2}}{x^2}\cdot\frac{2x\sqrt{1-x^2}-x^2\cdot\frac{-2x}{2\sqrt{1-x^2}}}{1-x^2} = \frac{\sqrt{1-x^2}}{x^2}\cdot\frac{2x\sqrt{1-x^2}+\frac{x^3}{\sqrt{1-x^2}}}{1-x^2} = \frac{\sqrt{1-x^2}}{x^2}\cdot\frac{2x-2x^3+x^3}{(1-x^2)\sqrt{1-x^2}} =$$

$$= \frac{1}{x^2}\cdot\frac{2x-x^3}{(1-x^2)} = \frac{2-x^2}{x(1-x^2)}$$

u) $y = \left(\frac{1+\text{sen}x}{\cos x}\right)^2$

$$y' = 2\cdot\left(\frac{1+\text{sen}x}{\cos x}\right)\cdot\frac{\cos^2 x-(1+\text{sen}x)(-\text{sen}x)}{\cos^2 x} = 2\cdot\left(\frac{1+\text{sen}x}{\cos x}\right)\cdot\frac{1+\text{sen}x}{\cos^2 x} = 2\left(\frac{1+\text{sen}x}{\cos x}\right)^2$$

v) y = -xctg x + ln (sen x)

$$(\text{ctg } x)' = \left(\frac{\cos x}{\text{sen}x}\right)' = \frac{-\text{sen}^2x-\cos^2 x}{\text{sen}^2x} = -1-\text{ctg}^2x = \frac{-1}{\text{sen}^2x}$$

$$\text{Ln (sen x)}' = \frac{\cos x}{\text{sen}\,x} = \text{ctg}\,x$$

$y' = -\text{ctg } x + x(1 + \text{ctg}^2 x) + \text{ctg } x = x(1 + \text{ctg}^2 x)$