

Resuelve las siguientes ecuaciones bicuadradas:

a) $x^4 - 5x^2 - 36 = 0$

$$y^2 - 5y - 36 = 0$$

$$y = \frac{5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot (-36)}}{2 \cdot 1} = \frac{5 \pm 13}{2} \Rightarrow \begin{cases} y_1 = \frac{5+13}{2} = 9 \Rightarrow y_1 = 9 \Rightarrow \boxed{x = \pm 3} \\ y_2 = \frac{5-13}{2} = -4 \Rightarrow y_2 = -4 \end{cases}$$

b) $x^4 - 4x^2 + 3 = 0$

$$y^2 - 4y + 3 = 0$$

$$y = \frac{4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{4 \pm 2}{2} \Rightarrow \begin{cases} y_1 = \frac{4+2}{2} = 3 \Rightarrow y_1 = 9 \Rightarrow \boxed{x = \pm \sqrt{3}} \\ y_2 = \frac{4-2}{2} = 1 \Rightarrow y_2 = 1 \Rightarrow \boxed{x = \pm 1} \end{cases}$$

c) $x^4 - 5x^2 + 4 = 0$

$$y^2 - 5y + 4 = 0$$

$$y = \frac{5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{5 \pm 3}{2} \Rightarrow \begin{cases} y_1 = \frac{5+3}{2} = 4 \Rightarrow y_1 = 4 \Rightarrow \boxed{x = \pm 2} \\ y_2 = \frac{5-3}{2} = 1 \Rightarrow y_2 = 1 \Rightarrow \boxed{x = \pm 1} \end{cases}$$

d) $x^4 + 16 = 0$

$$x^4 = -16 \Rightarrow \boxed{\text{No tiene solución real}}$$

e) $x^4 - 12x^2 - 64 = 0$

$$y^2 - 12y - 64 = 0$$

$$y = \frac{12 \pm \sqrt{12^2 - 4 \cdot 1 \cdot (-64)}}{2 \cdot 1} = \frac{12 \pm 20}{2} \Rightarrow \begin{cases} y_1 = \frac{12+20}{2} = 16 \Rightarrow y_1 = 16 \Rightarrow \boxed{x = \pm 4} \\ y_2 = \frac{12-20}{2} = -4 \Rightarrow y_2 = -4 \end{cases}$$

f) $x^4 - 81 = 0$

$$x^4 = 81 \Rightarrow x = \pm \sqrt[4]{81} = \pm 3 \Rightarrow \boxed{x = \pm 3}$$

g) $9x^4 - 8x^2 - 1 = 0$

$$9y^2 - 8y - 1 = 0$$

$$y = \frac{8 \pm \sqrt{8^2 - 4 \cdot 9 \cdot (-1)}}{2 \cdot 9} = \frac{8 \pm 10}{18} \Rightarrow \begin{cases} y_1 = \frac{8+10}{18} = 1 \Rightarrow y_1 = 1 \Rightarrow \boxed{x = \pm 1} \\ y_2 = \frac{8-10}{18} = -\frac{2}{18} \Rightarrow y_2 = -\frac{1}{9} \end{cases}$$

h) $x^4 + 21x^2 - 100 = 0$

$$y^2 + 21y - 100 = 0$$

$$y = \frac{-21 \pm \sqrt{21^2 - 4 \cdot 1 \cdot (-100)}}{2 \cdot 1} = \frac{21 \pm 29}{2} \Rightarrow \begin{cases} y_1 = \frac{-21+29}{2} = 4 \Rightarrow y_1 = 4 \Rightarrow \boxed{x = \pm 2} \\ y_2 = \frac{-21-29}{2} = -25 \Rightarrow y_2 = -25 \end{cases}$$

i) $x^4 + x^2 + 8 = 0$

$$y^2 + y + 8 = 0$$

$$y = \frac{-1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} \Rightarrow \boxed{\text{No tiene solución real}}$$

j) $x^4 + 3x^2 - 4 = 0$

$$y^2 + 3y - 4 = 0$$

$$y = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{-3 \pm 5}{2} \Rightarrow \begin{cases} y_1 = \frac{-3+5}{2} = 1 \Rightarrow y_1 = 1 \Rightarrow \boxed{x = \pm 1} \\ y_2 = \frac{-3-5}{2} = -4 \Rightarrow y_2 = -4 \end{cases}$$